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AN EXPERIMENT: THE TEACHING OF HIGH SCHOOL PHYSICS IN SEGREGATED CLASSES.[1]

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As was indicated in the report upon fundamentals given by Professor Galloway yesterday, our far-sighted educators are emphasizing in these days the *motivation* of the high school course of study.

This is leading to a careful scrutiny of the incentives and interests of the pupils in our science classes, particularly so by teachers of physics; in this subject, we have as students two groups of individuals, boys and girls, who come to our classes with different experiences, interests, and anticipations. If physics is to have its proper motivation for these two distinct groups, some differentiation in *presentation* or in *subject matter* is indicated to the educational practitioner.

In many schools, the number taking physics is not large enough to justify the formation of separate classes for boys and girls. In our larger schools where it could be done, while many have considered the proposition, some would without doubt question the desirability of such a procedure. For as a part of our educational inheritance from the past has come the idea that all pupils in a given subject should have the same training, so that all may be equally fitted for college or for life.

This idea of the same training for all is being seriously questioned in these days, when we are learning of the great variation in individual capacities even in such selected groups as high school and college classes.

We are at the dawn of a new era in educational progress in high school instruction in which the *pupil* rather than the *subject studied* is to receive the greatest consideration and when

[1]Read before the Physics Section of the Central Association, at Cleveland, Ohio, Nov. 26, 1910.

we see the necessity of going down to a pupil's level and skillfully leading him up to some of the mountain peaks of scientific thought rather than sitting in state and issuing directions for the journey over the *course* that we have previously traveled.

We are learning to ask, What are our pupils' real needs? and can we satisfy them by our teaching?

It is a common observation of thoughtful teachers of physics that marked differences exist between boys and girls in their stock of experiences and ideas, that are of service in the subject of physics.

Boys have many opportunities to learn about machines, electricity, properties of matter and illustrations of the laws of motion that rarely come to the average girl, especially to the girl born and reared in a city. It is a matter of common experience that in high school classes in physics, girls are often timid about asking questions especially on topics in which boys are apt to be proficient, fearing to expose their ignorance, or perhaps fearing more the smiles which arise when their inexperience leads them to ask foolish questions. Some consideration of these facts led me some years ago to ask permission of my principal to try as an experiment teaching physics in segregated classes. Approval for this plan was doubtless facilitated by the fact that in the Englewood High School boys and girls are taught in separate classes during the first two years. Several advantages are found in this plan which keeps the boys and girls apart in their recitations in the first two years of the high school, at a time when the girl is ahead of the boy in mental development.

Permission having been secured, a start was made in February, 1909, when at the beginning of the second semester a girls' class, a boys' class and a mixed class were arranged. In order to make the test as scientific and complete as possible it was planned to conduct the experiment in two stages. In the first, the instruction in all the classes was to be made as nearly alike as possible and, after the results of such a course were obtained, to take up the second part and present a course somewhat modified, giving boys more mechanics and electricity and girls more heat, sound, and light than would be assigned to the usual mixed class.

The prosecution of this experiment has not as yet passed beyond the first stage as outlined above. We are still testing results from teaching the same physics, i. e., the same topics

and the same laboratory work to all classes. I had planned to change to the second step this year, but being engrossed with a second pedagogical experiment which involves a shifting of the order of topics in mechanics to one that seems to me a more natural development of the subject in the mind of the average high school pupil, I am still presenting the subject as nearly alike to all pupils as possible.

In the first place, one finds that it is practically an impossibility to teach physics to a girls' class just as you would teach it to a boys' or to a mixed class. So that there is but little question that despite care on my part to prevent it, I have been presenting in a way three different courses. This is indicated by the fact that a boys' class in physics can cover as a rule ten to twenty-five per cent more ground in a recitation period than a girls' or mixed class, especially in mechanics. This is due in part to the previous experience of the boys, giving them many practical ideas, that must be developed in a class containing girls.

Another result of my observations has been that there is much more *unity* in a boys' or girls' class than in a mixed class. That is, the individuals in a segregated class have more nearly the same ideas, experiences, and interests, and you can appeal more successfully to the class as a whole by a proper selection of illustrations and experiments.

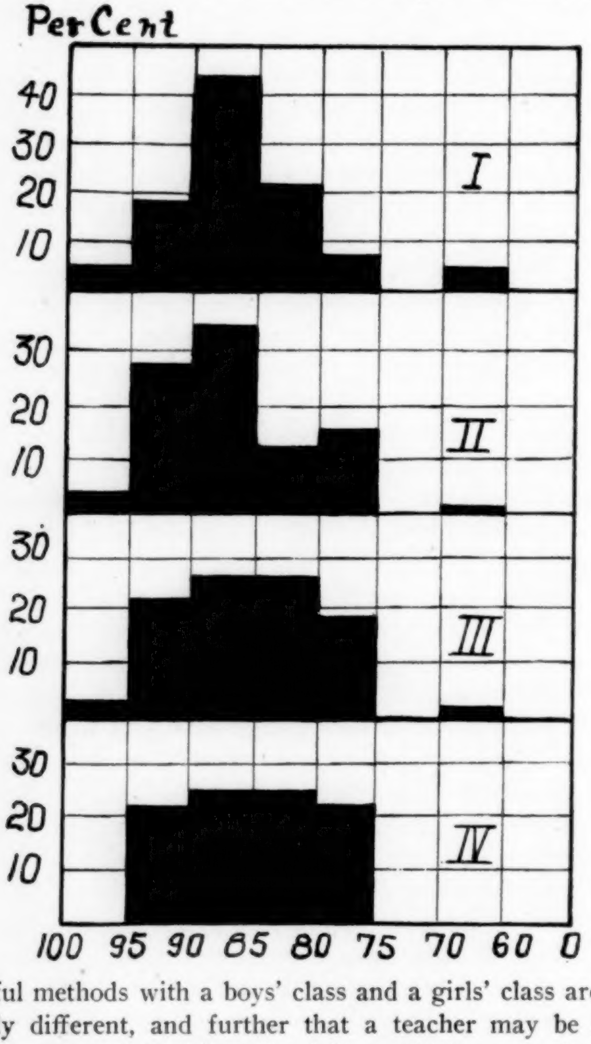
As an illustration of this idea, the principal of the school, Mr. Armstrong, was invited to give a lecture on some topics in sound to the three classes. Among other points, that of the tempered scale as applied to instruments with fixed tones was mentioned. The girls' class was intensely interested and raised numerous questions showing clear comprehension because of their interest in and knowledge of music.

The boys' class listened respectfully but no questions were heard.

In the mixed class but two or three girls asked questions.

Our school records for last year contain the results of teaching physics in segregated and mixed classes. The final averages attained for the year have been tabulated and graphs constructed which give the percentages of the pupils reaching certain grades. These graphs have been run off on the mimeograph and will be given to you at this time. The graphs show no marked differences between the several classes. In fact, the similarity between the several graphs is striking and as far as

the statistics go, it apparently has made but little difference whether the pupils were in mixed or segregated classes. This was not unexpected. It may be partly explained by a discovery made through the experience with segregated classes in the first two years. It has there been brought out strongly that



successful methods with a boys' class and a girls' class are often markedly different, and further that a teacher may be a conspicuous success with girls and a failure with boys or vice versa, and that his success with mixed classes is no certain criterion of his results with segregated ones. The same methods, topics, etc., having been used in all the classes no striking differences were expected, in the first part of the experiment.

We will recall that physics is one of the studies that appeals more to boys than to girls. The boys show more interest in it and are often the more successful students. I believe that this is due, not only to the subject matter but also to the method of presentation. It has usually been presented from the point of view of the boy, and in it have been emphasized some topics which may later be of use to the technical and to the research student though of doubtful value to the majority of our pupils. Some recent texts have shown that these points are being given thoughtful attention by their authors, though a text in physics adapted to the girls has yet to be written.

I asked my girls' class the other day whether they had a preference between a segregated and a mixed class in physics. Out of 29, 24 favored the segregated, two the mixed, and three were indifferent.

In a boys' class of twenty-six, twenty-one favored the segregated, one the mixed, and four were uncertain.

The same question raised in my mixed class brought a different answer. Of fourteen girls, ten favored a mixed class, four the segregated, while of nine boys, three favored the mixed, six preferring the segregated.

Of the reasons advanced by the girls, one especially interested me: "In a girls' class, the girls have to do their own work, and do not depend upon the boys."

The boys like to get into the boys' class where, as they say, there are no girls to hold them back. On the other hand a majority of the girls prefer the girls' class where they feel that they can ask questions and express themselves more freely. Some of the girls say they like the mixed class because they receive the benefit of the boys' questions and ideas.

Some consideration is to be accorded this point. In fact, it is the one most often mentioned by the girls in favor of the mixed classes. Its value, however, is more apparent than real, as its application frequently permits the stronger members of the class to do the thinking for the weaker or less experienced ones, whereas these latter ones are those most in need of exercising their reasoning powers, and these will develop best when these pupils are placed where they are obliged to do their own thinking.

The general results thus far show: 1st. That teaching physics in segregated classes is fully as successful as in mixed ones.

2d. The plan is favored by eighty per cent of those in the segregated classes.

3d. The boys progress faster and enjoy the work better alone.

4th. The girls express themselves better and more fully when by themselves.

5th. The teacher gets more mental exercise than usual in adapting methods to the situation and sometimes gets out of the "ruts" in which he has been traveling.

In conclusion, this experiment has led me to see certain decided advantages in segregated classes in physics. We plan to continue the experiment through the second stage in which the courses will be somewhat modified.

The Physics Section of the Central Association of Science and Mathematics Teachers has appointed a committee to study the results of teaching physics in segregated classes and to report at the next meeting of the Central Association. All teachers conducting segregated classes in Physics are therefore urged to communicate with the chairman of the committee, Willis E. Tower, Englewood High School, Chicago, Ill.—ED.

SCIENCE NEEDS REORGANIZATION.

The route by which a goal is attained is as important as the goal to be attained. We need a reorganization of science in the high school for the need of the adolescent boys and girls.—President Keith, Oshkosh State Normal.

MEASURE THE TEACHER AND HIS WORK.

The efficiency of public schools requires that the conditions and results of the teachers' work be subjected to definite standards of measurement. We are in need of mathematics rather than rhetoric in order to properly appreciate the degree of success to which a teacher or school attains.—Professor E. C. Elliot, University of Wisconsin.

WATER POWERS OF THE CASCADE RANGE IN SOUTHERN WASHINGTON.

Water powers in the Far West have attracted general public attention during the last few years, and their utilization and protection from monopoly have been subjects of wide discussion and of important executive action. Perhaps no area in that region presents more favorable opportunities for water power development than the slopes of the Cascade Range. The general elevation of the range is six thousand to eight thousand feet. The streams draining it have steep grades and are fed during the period of low water by snow banks and glaciers or by copious supplies of ground water. The three important requisites of water power—rapid fall, abundant water, and comparative uniformity of flow—are found in these streams, and the great resources of forests, mines, and soil in the region offer fair promise of a good market for water power.

BERNOULLI'S PRINCIPLE.

By William S. Franklin,
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When water flows out of a tank (where the pressure is high) into the air (where the pressure is low), the water gains velocity. This is a very familiar fact. The water in the jet moves faster than the water in the tank!

Water gains velocity when it moves from a high-pressure region to a low-pressure region; and the converse is also true, namely, *whenever water has gained velocity it has moved from a high-pressure region to a low-pressure region* (Bernoulli's Principle). Let it be understood that the effects of gravity are not considered. A loss of velocity, however, does not always mean that the water has moved from a low-pressure region into a high-pressure region, because velocity may be lost by friction. When, however, the effects of friction are negligible, then it is true that *whenever water has lost velocity it has moved from a low-pressure region to a high-pressure region* (Bernoulli's Principle). It must be remembered that we are not here concerned with downhill or uphill flow, the flow is understood to be horizontal.

When water flows along a horizontal pipe of variable section, the velocity of the water is small where the pipe is large, and the velocity of the water is large where the pipe is small; and *where the velocity is large the pressure is small and where the velocity is small the pressure is large* (Bernoulli's Principle). A clear understanding of these facts may be reached on the basis of Newton's second law of motion as follows: Consider a piece of water w as it passes through a throat in a pipe as indicated in Fig 1. This piece of water moves at low velocity when it is at A, at high velocity when it is at B, and at low velocity again when it is at C. That is to say, the piece of water w gains velocity while it is moving from A to B, and it loses velocity while it is moving from B to C. Therefore, according to the second law of motion, an unbalanced force must be pushing forwards on the piece of water while it

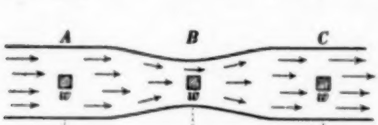


Fig. 1.

is moving from A to B, and an unbalanced force must be pushing backwards on the piece of water while it is moving from B to C. Now an unbalanced force pushing backwards on the piece of water under consideration means that the pressure of the water in the region ahead of *w* is greater than the pressure of the water in the region behind *w*. This condition exists as long as the piece of water under consideration is losing velocity, that is, while the piece of water is moving from B to C. Therefore the pressure in the region C is greater than the pressure in the region B. A similar argument shows that the pressure at A is greater than the pressure at B.

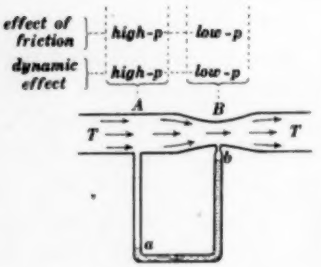


Fig. 2.

An interesting demonstration of Bernoulli's Principle may be given by means of the device shown in Figs. 2 and 3. Air is blown through the pipe TT, and the difference of pressure between the places A and B and between the places B and C depends upon two effects, namely, the friction effect and the dynamic effect (Bernoulli's effect). That is to say, the pressure of the air in the pipe grows less and less in the direction of motion of the air because of friction; and the pressure is small where the velocity is large and large where the velocity is small, according, to Bernoulli's Principle. In Fig. 2 the friction effect and the dynamic effect *both* tend to produce a lower pressure at B than at A as indicated by the difference of level in the arms of the attached manometer *ab* as shown in the figure. One might think, therefore, of the difference of level in the arms of the manometer as being wholly due to the familiar friction effect. In Fig. 3, however, the effect of friction tends to produce a lower pressure at C than at B, and the dynamic effect is much the greater of the two as indicated by the attached manometer *bc*. In this case the dynamic effect stands out prominently because it is opposite to the friction effect and sufficiently great to show itself in spite of the friction effect.

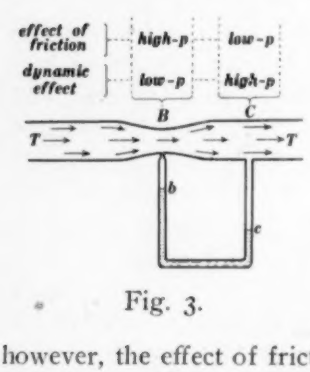


Fig. 3.

If one blows a very strong blast through the pipe TT in Fig. 3, the water in the manometer *bc* will be drawn up into the throat and carried forwards with the jet of air in the form of spray. This action is exemplified in the familiar device called the *atomizer*.

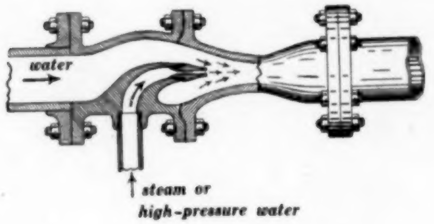


Fig. 4.

The diminution of pressure in a throat as shown in Fig. 3 is utilized in the familiar jet pump and in the steam-boiler injector. Fig. 4 shows a sectional view of a simple form of a jet pump.

An Example of Bernoulli's Principle and an Analogy Thereto.

Actual Example.

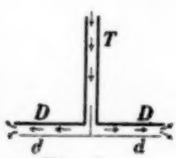


Fig. 5.

Air is blown through the tube T and spreads outwards between the disks DD and *dd* as shown in Fig. 5.

The disk *dd* is prevented from moving sidewise by a pin which projects into the end of the tube T.

The air between the disks is moving outwards at high velocity, and it is at lower pressure than the still, outside air.

As the air between the disks reaches the edge of the disks it loses its velocity, and in losing its velocity it pushes itself into the high-pressure outside air.

Inasmuch as the pressure of the air between the disks is less than the pressure of the outside air, it is evident that the disk *dd* is pushed upwards by the outside air with a

Analogous Case where Differences of Level Take the Place of Differences of Pressure.

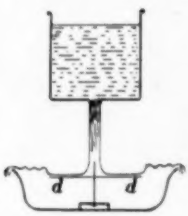


Fig. 6.

Water falls in a jet and spreads outwards upon a disk *dd* as shown in Fig. 6.

The disk *dd* is prevented from moving sidewise by a pin which projects through a very small hole in the disk.

The water on the disk *dd* is moving outwards at high velocity and it is at a lower level than the still water in the basin.

As the water on the disk reaches the edge of the disk it loses its velocity, and in losing its velocity it raises itself to the level of the water in the basin.

Inasmuch as the water on the disk *dd* is at a lower level than the water in the basin, it is evident that the disk *dd* is pushed upwards by the water in the basin with a greater

greater force than it is pushed downwards by the air between the disks.

In fact the disk *dd* is held up by blowing through the tube T.

force than it is pushed downwards by the water on top of it.

In fact the disk *dd* is made to float by the jet which falls upon it.

An interesting example of Bernoulli's principle is afforded by the floating of a light ball in a jet of air or steam. Such a floating ball darts about continuously, and whenever it gets part way out of the jet, as shown in Fig. 7, the greater pressure of the outside (still) air pushes it back into the jet again, as indi-

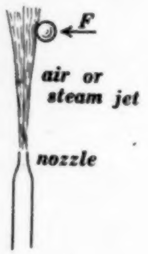


Fig. 7.

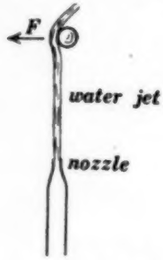


Fig. 8.

Fig. 9.

cated by the arrow F. A ball will float on a water jet in the same way, but in this case it is not simply the Bernoulli effect that keeps the ball in the jet. When the ball is in the position shown in Fig. 8, the jet adheres to the ball and is thrown to one side as shown; the ball pulls the jet to one side and the jet pulls the ball in the direction of the arrow F. This effect is exemplified in a very striking way by placing the finger against the side of the jet of water from a hydrant, as shown in Fig. 9.

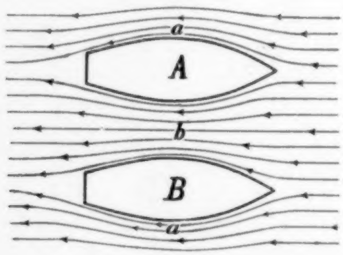


Fig. 10.

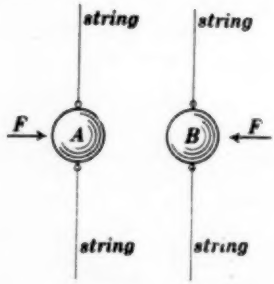


Fig. 11.

The finger is pulled in the direction of the arrow F with a very considerable force.

An interesting effect depending upon Bernoulli's Principle is

that two ships steaming along side by side are pushed towards each other by the water. Thus when a squadron of warships is maneuvering it is dangerous for one ship to attempt to pass close by another. In order to understand this effect it is helpful to consider the equivalent case of two ships standing still side by side in a stream which flows swiftly past them, as shown in Fig. 10. The velocity of the water is much greater in the region *b* between the ships than it is in the outside regions *aa*, and therefore the pressure in the region between the ships is less[1] than the pressure in the regions *aa*, so that the ships are pushed towards each other. An effect similar to the attraction of two ships in a stream may be shown by suspending two light

Fig. 12.

Fig. 13.

balls side by side in the blast of air from an electric fan, as indicated in Fig. 11. The blast pushes the balls together.

Fig. 12 shows how a stream of liquid or gas flows around a flat plate PP. The velocity of the fluid is great at the point *a*, and small at the point *b*. Therefore the pressure of the fluid is great at *b* and small at *a*. This difference of pressure turns the plate in the direction of the curved arrows *cd*, thus bringing the plate into a position with its plane at right angles to the stream. This effect can be shown by placing a pivoted disk in the blast of air from an electric fan, or it can be shown by allowing a flat sheet of paper to fall through the air. The manner in which such a sheet of paper falls is indicated in Fig. 13. A piece of paper with its edges turned up falls as indicated in Fig. 14 without darting to and fro sidewise. This difference in the behavior of a plane surface and a curved surface in a current of air is exemplified by the difference in the behavior of the old-fashioned paper kite with a flat surface and the more recent

[1]The water level between the two ships is lower than the normal water level of the stream.

type of kite with a curved surface. The old-fashioned kite tends to dart to and fro sidewise, and it has to have a tail to counteract this tendency; whereas the kite which has a curved surface flies very quietly and without a tail.

The wing surface of an aëroplane is never flat as shown in Fig. 15, but curved as shown in Fig. 16. If the wing surface were flat most of the upward force would be exerted at the forward edge *b* as shown by the heavy arrow F and as explained

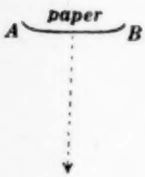


Fig. 14.

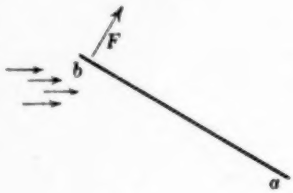


Fig. 15.

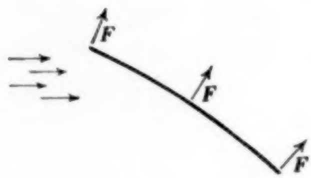


Fig. 16.

in connection with Fig. 12. When the wing surface is curved as shown in Fig. 16 an approximately uniform upward pressure is exerted over the whole surface, as indicated by the arrows FFF in Fig. 16.

The curved flight of a spinning ball, as exemplified by the curve of an expert baseball pitcher, is an example of Bernoulli's Principle. To understand this effect it is best to consider the case of a spinning ball which stands still in a stream of air, instead of considering a spinning ball which moves forwards through still air, the two case^s being exactly equivalent to each

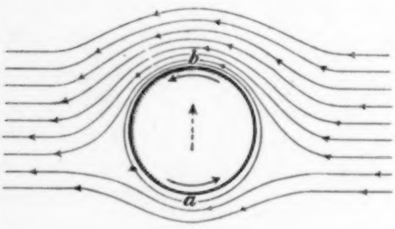


Fig. 17.

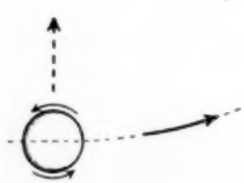


Fig. 18.

other. Thus Fig. 17 shows the manner in which a blast of air flows past a spinning ball. The spinning motion of the ball causes the air blast to be turned to one side of the ball as shown, and the velocity of the air at *b* is much greater than the velocity of the air at *a*, therefore the pressure of the air at *a* is greater

than the pressure of the air at *b*, so that the air pushes the ball in the direction of the dotted arrow.

Figure 17 represents a spinning ball in a stream of air which flows to the left, and the effect is the same as if the spinning ball were moving towards the right through still air as shown in Fig. 18. The spinning motion of the moving ball in Fig. 18 causes the air to push sidewise on the ball in the direction of the dotted arrow, thus causing the ball to describe a curved path.

The curved flight of a spinning ball may be shown very beautifully by throwing a ball of cork or pith by means of the device shown in Fig. 19. A fine thread is tied to the end of a light rubber band and wrapped around the ball. The rubber band is then stretched, and when the ball is released it is thrown for-

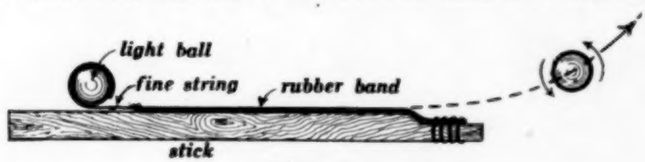


Fig. 19.

wards with a spinning motion, as indicated by the curved arrows in Fig. 19. This spinning motion causes the ball to be pushed upwards by the air as explained in connection with Fig. 18. A light ball may thus be made to curve upwards to the ceiling of a room, although the initial direction of its motion is horizontal.

The motion of a "high foul" is an interesting example of the curved flight of a spinning ball. The ball strikes the bat as indicated in Fig. 20, and rebounds in the direction of the arrow I. At the same time the ball is set spinning in the direction of the small curved arrows. On account of this spinning motion the air pushes the ball in the direction of the arrow U on the upward flight and in the direction of the arrow D on the down-

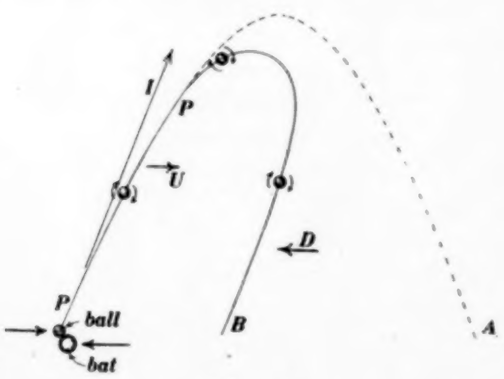


Fig. 20.

ward flight, as explained in connection with Fig. 18. The catcher, judging from the early portion PP of the ball's flight, runs to point A expecting to catch the ball, but the ball turns inwards to the point B. This failure of a catcher in the case of a "high foul" is sometimes very amusing.

The curved flight of a "high foul" can be shown by throwing a small ball of pith or cork, or best of all an oak gall, into the air in a manner similar to the shooting of a marble so as to cause the ball to spin as indicated in Fig. 20.

RESULT OF EXPERIMENT TO DETERMINE CONTENT AND APPEAL OF FIRST YEAR SCIENCE.

BY FAITH MCAULEY,
High School, St. Charles, Ill.

For two years we have been giving a course in elementary science at St. Charles. The introduction of this work was prompted by several things, foremost of which was the lack of the scientific habit of approach to a topic, evident in the second year pupils beginning botany or zoölogy; second, the feeling that there is fundamentally important material which should be the possession of each student irrespective of subsequent specialization; third, the conviction that such fundamental material would act as a natural gateway to subsequent work such as effective physiology, intelligent physiography, etc.

Without discussing the content or unifying element in the work, I shall give briefly, the results of two experiments.

By the first experiment we hoped to be helped toward an answer as to the suitability of the material chosen. The introductory work is based on air and water and so is largely chemical. In order to determine whether this material was as workable with first year pupils as with third, parallel examinations were given to the elementary science and chemistry classes, five out of the ten questions being the same for each group. Below is the result based on the five questions.

	No. in Class	Average Grade Scale of 50	Per Cent Passing the 5 Questions
Chemistry	25	31.68	40
Elementary Science . . .	42	33.09	40.4

The above data would seem to indicate that the material chosen is as well adapted to the first year as to the third.

Second, wishing to be guided in formulating the work by the attitude of the students themselves, the class of last year was given the following questions:

1. Which subject in first year required most effort?
2. Which do you think was most profitable?
3. In which were you most interested?

	Quest. 1 Effort	Quest. 2 Profit	Quest. 3 Interest
*Algebra	4	2	7
*English	5	4	—
Latin	5	7	4
*Elem. Science	11	9	9
Physiography	—	—	2
Drawing	1	4	4
No. in Class 26			
*Required Subjects			

The above results seem to me to suggest that elementary science may rightly have a place in first year work.

ASBESTOS IN THE UNITED STATES.

An International Industry.

The United States has for years led all other countries in the manufacture of asbestos goods, but until recently all the raw asbestos used has been imported from Canada, where there are nineteen quarries and mills, having a capacity of 8,250 tons of rock a day and employing in summer more than three thousand persons.

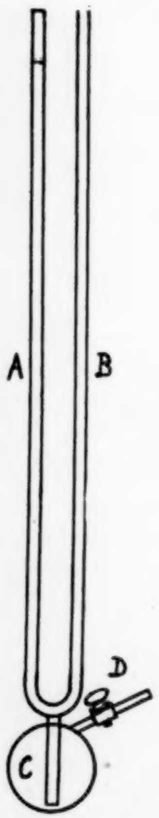
A feature of the asbestos industry of 1909 was a combination of Canadian producers in the Amalgamated Asbestos Corporation (Ltd.) and the formation of the International Asbestos Association, an organization including mine owners in Canada and manufacturers in the United States.

No asbestos of the higher grade (serpentine asbestos, or chrysotile) was mined in the United States until 1908, but in that year Vermont produced some chrysotile and in 1909 mined a larger quantity, amounting to nearly one twentieth of the Canadian output. Chrysotile asbestos has been mined in small quantities in Wyoming during the present year.

BOYLE'S LAW APPARATUS.

By E. J. Rendtorff,
Lake Forest Academy, Lake Forest, Ill.

There are many forms of Boyle's law apparatus. Most of them are open to various rather serious objections. They resolve themselves into two typical classes: those where the two tubes are connected by a rubber tube and the old-fashioned J form, where mercury must be added consecutively to the open tube in order to get a series of readings.



The first form always springs a leak in the rubber tube. The sliding attachments frequently become corroded and the reading scale is seldom conveniently placed. In the second form the mercury soon becomes impure, badly discoloring the tubes. There is also the danger of spilling the mercury over the surrounding apparatus.

Desirable features in a Boyle's law apparatus are:

1. Elimination of all rubber tubing between the open and closed tubes.
2. No undue exposure of the mercury to dust or other sources of impurity.
3. Scales placed in immediate contact with the tubes.
4. Elimination of all pouring of mercury into the apparatus to obtain a series of readings.

A very satisfactory form of apparatus is shown in the diagram. The closed tube A and the open tube B are each about 1 m. long and 8mm. in diameter. They join below in a tube that extends to the bottom of a bulb *c*, some 5cm. in diameter, to which is attached a stop cock D. An ordinary bicycle pump is attached to D by means of a short piece of rubber tubing. The bulb is filled with mercury.

The air is compressed until the mercury almost fills the open tube. Readings of the volume of the compressed air and the difference in height of the mercury columns are then made. By momentarily opening the stop cock the mercury is slightly lowered and another group of readings made. A series of ten readings can thus be easily obtained.

The apparatus is mounted on a suitable board and fastened to the laboratory wall. A meter stick just fits between the tubes. A guard passes over the lower bulb and the bicycle pump is rigidly attached, so that no great strain can come on the stop cock tube. A small piece of cotton batting placed over the open tube prevents the admission of dust.

This was found to be the cleanest, most rapid, and on the whole most satisfactory form of Boyle's law apparatus that the writer has ever used.

PHYSICAL GEOGRAPHY VERSUS GENERAL SCIENCE.

By C. R. Mann,
The University of Chicago.

Those interested in the problem of first-year science in the high school will read the article by Mr. H. W. Fairbanks in School Science and Mathematics for December with relish. As an ardent believer in a required course in general science in the first year of the high school, I cannot refrain from pointing out that Mr. Fairbanks's claim to have proved conclusively the futility of the general science idea, and to have shown the preëminent fitness of physical geography for the first-year course, is not at all substantiated by his arguments. In fact, his paper proves, if it prove anything, that physical geography is as ill-fitted as it could possibly be for this purpose, and that only a general science course can ever satisfactorily meet the present demand.

After stating that physical geography has failed to do its work because of "wrong ideas as to what physical geography should be, and because of the wrong attitude of most of the texts, and because of wrong conceptions of laboratory work," and after stating that the "conceptions behind general science are illogical," he says that general science and geography are closely related as to subject matter, but in treatment and viewpoint are as far apart as the east is from the west.

The distinction between the two methods of treatment is then described more fully by saying that the child has the "geographic" viewpoint, when he becomes interested in the various phenomena of the world as they occur in their natural relations; but that he is studying science for its own sake, when he begins to analyze the orderly aggregate of phenomena, and to separate

out related facts, and organize them in a formal and abstract manner. The "geographic" viewpoint is further defined as "the study of the earth as an organism, and of the various facts, biological, chemical, and physical, which go to make up its complex existence." Again: "General science takes up the materials as facts and leaves them as facts, while geography weaves them into a connected whole—the world in which the pupil lives." Finally we are told that "general science is essentially geographic, with the geographic concept left out" (whatever that may mean).

Mr. Fairbanks points out that "the rise of the general science idea is a protest against the formal and lifeless teaching of physical geography;" and then jumps to the very surprising conclusion that "with physical geography as a required subject and reasonably well taught, there is not the shadow of an excuse for the presence of general science in the high school." Thus, having first shown the failure of physical geography as a first-year science, and then having demolished to his own satisfaction the claims of general science because "it is essentially geographic with the geographic concept left out," and having set up the geographic ideal of a vital study of surrounding phenomena, "biological, chemical, and physical," in their natural surroundings, the conclusion is drawn that a return to the acknowledged failure of the required work in physical geography is the one and only solution of the problem!

The conclusion that follows logically from the argument is that a course in the scientific study of phenomena in their natural surroundings, with its parts organized about some central idea, and treated from what he defines as the "geographic" viewpoint is what is needed. In this conclusion I would heartily concur, since this is the ideal of "general science." The use of the word "geographic" to describe this idea is unfortunate, since the word is commonly used in a different sense. The final use of the word "physical geography" as synonymous with "geographic" in this peculiar sense tends to befog rather than to make clear the real issue. The battle is not the sham battle between physical geography and general science, as Mr. Fairbanks would have us believe; but a real battle between a vital, concrete, significant, worth-while study of science—not only in the first year, but all through the course—and the formal, abstract, logical, coldly intellectual system of science whose teaching is now attempted in most schools.

The real contest is thus not one between rival systems of classifications of science, but is one between "formal discipline," on the one hand, and "motivation," on the other. The real question involved is whether it is possible to develop character by drilling pupils in an effort to make them learn a logically arranged and purely intellectual system of thought for the purpose of attaining some distant and vaguely perceived end; or whether the emotions and feelings of the pupils during the process must be taken into account, and the effort made to foster such motives in them as will impel them to work because they appreciate the value and the significance of the work. In other words, the fundamental question is whether we are going to try to select and present the materials of science in such a way that science will become significant to the pupils, in the hope that they will then work at it for all they are worth under the spur of their own desire to master it; or whether better results are gained by attempting to teach formal systems of science, thereby driving the pupils through a drudgery which we justify to them by telling them that everybody has to do disagreeable things some time, and the sooner they learn to do them pleasantly from a sense of duty and a fear of authority, the better for them.

Solutions of alum have long been employed for intercepting the calorific rays of light. The use of alum for this purpose was an article of faith until an experimenter discovered that pure water possesses quite as much absorbent power as a solution of alum. Houston and Logie have been seeking a still more powerful absorbent of thermal rays, which is required in many cases, and have found it in a solution of ammoniacal sulphate of iron. A stratum of this solution, 1.2 inches in thickness, transmits 75 per cent of the luminous rays, but only 5.1 per cent of the total radiation of a carbon filament electric lamp. It should be added that the luminous rays constitute only about 2 per cent of the total radiation, in this case.

The latest addition to the world-wide campaign of upper-air research, is the work in this line just begun at Cordoba, Argentina, under the direction of the meteorological service of that country. The work was organized by Mr. H. H. Clayton, late of the Blue Hill Observatory. An aerological staff of five observers is now making, so far as conditions permit, daily observations with kites, and will later undertake balloon observations. The aerological station at Cordoba is notable as the only institution of its kind in South America, and with the possible exception of Samoa, the only place in the southern hemisphere at which regular and frequent soundings of the upper air are in progress.

PHYSICAL CHEMISTRY—A BASIS FOR SECONDARY SCHOOL CHEMISTRY (?)[1]

By Vergil C. Lohr.

Perhaps it would have been well to have put the topic in this wise, "Can physical chemistry and the principles of physical chemistry be used to advantage in a secondary school course?"

We are all pretty well of the opinion, I believe, that the trend of scientific education of the past has been almost entirely an education of listening, absorbing, and appropriating, not of seeing and getting by activities, or as Dewey has put it, taught "too much as an accumulation of ready-made material with which students are to be made familiar, not enough as a method of thinking"—an attitude of the mind after the pattern of which mental habits are transformed; and we are of the opinion that a new education in science is upon us, which has for its goal that broader object—the making of a well-trained man not an anemic encyclopædia of facts. Yet many a so-called teacher, I fear is to-day following along the beaten path in the field of chemistry believing that he must teach the same things, after the same manner, as he was taught.

Why not challenge the assumption that seems to have been made in so many cases, that we have to teach in a more or less parrot-like fashion the properties, behavior, etc., of all the elements and their compounds? Since we cannot hope to teach all of chemistry in secondary schools, how much shall be taught, and what? This is the old question which has confronted us in such meetings for years, and one which I shall not attempt to answer. But are we not looking at the subject from the wrong angle? Rather than how much, shall we not turn our attention to the "How"?—How shall chemistry be taught? This problem properly solved will in a large measure solve the first.

At the outset let me say that I have no sympathy for "fads" or "cheap" science of any kind—painless science administered by painless methods is worse than useless and already far too prevalent.

Much less sympathy have I for that course which is injected with theory for the purpose *only* of gratifying the whim of some specialist—neither have I toleration for that course in any subject which has only for its excuse the meeting of some requirement imposed by some higher institution of learning.

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[1]Read before the Chemistry Section of the Central Association at Cleveland, Ohio, Nov. 25, 1910.

I am of the opinion, too, that one kind of chemistry will not meet the needs or requirements of all types of mind with which we have to deal. But that chemistry of whatever kind it is, should aim, not at cramming the head with crude facts, but at developing personal initiative a desire for investigation—at making individual creators. This can be done only by teaching principles, and clinching those principles in such a way that they will be recognized and used wherever met. A high school student finishing his first course in chemistry should be grounded good and deep in the basic principles of the science. If he is worth much, he will get the details of the subject at the same time; if he is worth little, these details may be lost, but the mental process has been awakened and the principles will remain. We do not expect to find trained scientists in secondary pupils, but that course in science which does not in a measure develop the scientific habit, has in that measure at least been a failure.

Now to the application of the subject. Do physical chemistry and the principles of physical chemistry offer a possible aid in the accomplishment of this somewhat idealistic ideal? Surely we cannot get far away from a physical basis for chemical manipulation for "Physical phenomenon is the language of chemistry." Change the viewpoint of much of chemistry and we have but one place to go, and that to physical chemistry. Place the emphasis on the *"WHY"* and where must we turn for the answer? From my point of view it is of far more value to know *WHY* magnesium chloride is deliquescent and glaubers salt is efflorescent and why blue vitriol is ordinarily neither the one nor the other, than to know by rote all the substances belonging to one class or to the other. It is a question of only a few quantitative vapour pressure determinations and a few well directed questions to teach the individual not *which* deliquesce but *why* they deliquesce. I am aware that the time element plays an important part in deciding upon such a course of action, but a marked growth in the ability of the pupil more than compensates in the end for any apparent loss of time in the beginning.

A few principles well learned is better by far than an accumulation of many unrelated facts.

I hear you say there is already too much to be taught, why add more? My answer is that principles well understood diminish the quantity, to illustrate—Teach well the principles of oxidation and you have made a short job of the halogens. How much of the student's time is really necessary to learn the physical

properties of the halogens or of HNO_3 if the attention is directed to the "how" and "why" of the fact and not to the fact alone? Shall we for example let the mere statement that chlorine is a bleaching agent pass, or shall we investigate how and why chlorine acts when bleaching is accomplished? Shall we teach merely that MNO_2 and $KMNO_4$ with HCl give Cl_2 and HNO_3 with S. give H_2SO_4? Or shall we teach the principle of oxidation and require the student to explain *Why* these things come to pass?

The trend of modern chemical pedagogy seems to be toward an abbreviated treatment of the chemical laws and elimination of so-called chemical theory. It may sound like heresy to say it, but I firmly believe that from the point of view of interest and development of "chemical sense" on the part of the student more of theory, illustrated by concrete examples, is desirable rather than less of it. With Le Chatalier's theorem in view the study of a very few solutions with respect to thermal and volume changes and a liberal use of solubility graphs the *why* of increased solubility with increased temperature in some cases and vice versa is easily disposed of. Is this not of more value than merely knowing that some substances belong to one category and some to another? Or even knowing in a perfunctory way which belong to which. The kinetic molecular hypothesis, solution pressure, vapor pressure, osmotic pressure, yes, even the phase rule all lend themselves to a complete understanding of questions of saturation, freezing mixtures, boiling point and kindred questions, and give the student a sane view of the unity of things.

Nor would I eliminate from a laboratory course the determination of some equivalent weights—three at least, with method so selected that the meaning of "equivalent weight" as correctly defined will be amply illustrated; nor eliminate exercises in determining molecular volumes for by such exercises is an easy insight into atomic weight attained. Define an atomic weight as the smallest weight of an element that occurs in a molar volume of all of the volatile compounds of the element and little difficulty presents itself.

The law of mass action may have no place in elementary chemistry, but what better basis for reversible reactions, chemical equilibria and displacement of equilibria? Shall this very important concept of chemical actions be omitted from a secondary course? It may be omitted, I grant, but the loss of that concept is

fatal to an understanding of hydrolization, dissociation, and ionization—Omit these and you are forced to omit a large and vital part of chemistry.

Is it not better that a pupil possess the ability to write the scheme of equilibrium for a solution of such as salt as $Na\ HCO_3$, or $Na_2\ HPO_4$ in water, and arrive at a complete and positive decision as to its acidimetry or alkalimetry, than by a memory process to have this information of all the salts in the kingdom? Is it not of fundamental importance to know why $Ca\ C_2\ O_4$ dissolves in HCl and not in $HC_2\ H_3O_2$, and why certain sulphides will precipitate in acid solution and others not? It is unquestionably desirable that a student know that $NH_4\ Cl$ prevents precipitation of $Mg\ (OH)_2$ from magnesium salt solutions by NH_4OH; but that student who has such a grasp on the subject that he can explain *why* no precipitate is formed is decidedly better equipped than one who knows the fact only. Is ionic product constant out of place in secondary chemistry? It may be from a theoretical point of view, but from actual experience I am of the opinion that it has a place and a fundamental place in such a course. Osomometer, boiling point and freezing point determinations with solutions of ionogens and non-ionic materials of various concentrations give clear ideas of the evidences of dissociation and head naturally to questions of ionization. A voltmeter and solutions of various concentrations make per cent of ionization a reality. The principles of ionization once illustrated, proper perspective gained, problems such as before mentioned become as nothing and are a source of constant delight to the normal pupil.

Nor have I spoken of things that are beyond the ability of the normal mind of the age and development at which they belong in chemistry. The conclusions I have reached in my own case are not ideas hatched from theory, but come from actual trial and observation for a period of three years during which time I have had ample opportunity to test its advisability and practicability. Of course the development and ability of the pupil must be constantly kept in mind, exhaustive treatment of physical chemical material cannot be attempted, nor is it desirable, only so much as is of service in making clearer some fundamental has a place. A keen balance must be kept between the theoretical and the practical by a skillful teacher who knows the subject and who is endowed with a fair degree of what might be termed "pedagogical sense."

In a brief way I have mentioned a few of the principles which

may be spoken of as physical chemical which have been found to be decidedly helpful in secondary chemistry. By their use one is enabled to teach more of elementary chemistry and better elementary chemistry than is possible without, and to send the pupil from the course equipped to meet a proposition squarely and to attack it in a logical sensible manner. The test of any course should be a certain power gained to do work, the ability to weigh evidence to discriminate between the probable and improbable—the real and the hypothetical.

I would not leave the impression that such a plan is in any way antagonistic to the report of your committee on fundamentals; I am in hearty accord with practically all of that report. But in the accomplishment of what is laid down as fundamental I have suggested some things which seem to me fundament for these fundamentals. I am not advocating the view that chemistry should be studied and taught for its intellectual content rather than for its material content. I believe that both demands can be satisfied simultaneously if they both are given the proper setting.

In the school with which I am associated the situation is in some respects perhaps unique. Chemistry is offered as an elective. Physics is a prerequisite and is required of all juniors. The city might be termed a manufacturing center and a large per cent of the boys who elect chemistry have industrial chemistry in view after leaving high school. These, I might say, at the time they come to us have been found to be among the strongest and best pupils of the school. A match factory, a wire mill, and the Illinois Steel Company have been able to use all whom we can recommend for positions in their laboratories. To meet the needs of others not so well prepared or who have other objects in view, and those wishing a general view of the subject, a second course is offered termed Household Chemistry, less intensive and employing domestic problems and domestic tests rather than the broader principles of the first mentioned course. Of seventy-five electing chemistry this fall, sixty elected the former—eight being girls, the fifteen electing the latter course are all girls. It is with the more intensive course that I have conducted my experiments and made my deductions. In this course the tendency is naturally to emphasize industrial processes and reactions and the pupil is viewing the subject from the standpoint of its commercial value. Since perhaps less than one-tenth of those who pass from this class into this industrial work will have opportunity for

further chemical preparation it behooves the high school to give to those pupils the very best possible training for that work. I can imagine a condition no more deplorable than to send such a pupil into such a sphere of activity with a perfunctory knowledge of chemical reactions and manipulations and without a broad view of the principles underlying such reactions and manipulations. Without the latter he must become a mere automaton, with no hope for future development or growth—as superficial as was his training. It was Goethe who said, "Nothing in the world is so terrible as activity without insight." In our zeal to "practicalize" and popularize chemistry I fear we are prone to allow the material content to eclipse all, and put a cheap value on the intellectual.

It is for the intellectual AND the material content, the theoretical AND the utilitarian that I plead, believing that the intellectual is the sane pathway to the material and that physical chemical principles is the fountain head of much of the intellectual. Does not Emerson's paradox apply to chemistry? Is not chemistry for the majority of high school students what remains to them after what they have learned in school is forgotten?

The recent electrical show held in Denver was one of the most successful exhibitions of its kind ever held in the West. The show was notable for its illumination not only within the auditorium, but outside as well. A novel feature of the outdoor illumination consisted of a representation of Franklin's kite soaring in the air. The kite was studded with electric lights and a string of small lamps led down to a large illuminated key. Within the auditorium a most unique spectacle was a sunrise picture. First the moon was shown sinking below the horizon, and as it faded from view the hills were lighted up by the rising sun, which was very realistically represented by 2,500 electric lamps. This illuminated picture occupied the whole of one side of the auditorium.

Large territories in Europe and Asia, under Turkish rule, abound in rivers that could be used to advantage to generate electricity. During the rule of Hamid II., such projects were out of the question; but now that the young Turks are in control of the government, electricity is gradually being introduced into the country, and plans are already on foot to utilize some of these resources that have heretofore been wasted. Our consul-general in Smyrna has recently called attention to a lake fifteen miles long, at an altitude of 2,500 feet above the surrounding plains, from which a stream of two thousand cubic feet per second flows during the winter season, while in summer time the flow is reduced to half this amount. It is estimated that three hundred thousand horse-power could be generated from this source. Not only could the power be used for generating electricity, but the water could be utilized to advantage for irrigating the arid plains.

SUGGESTIONS FOR A PHYSIOLOGICAL LABORATORY IN HIGH SCHOOLS.

By H. D. Densmore,

Beloit College.

One of the pressing needs of departments of botany in high schools, and in many colleges as well, is a room in which plants may be grown and experimented upon during the entire school year. The universities and some of the older colleges have greenhouses as accessories to their botanical departments in which fresh plant material of different kinds can be grown and prepared for class use. In such institutions a physiological laboratory independent of the growing houses is the logical and natural arrangement. In the high schools, however, such an arrangement is not feasible on account of the large expense

involved in maintaining a greenhouse and laboratory. A room combining some of the features of a growing house and an experimental laboratory is, therefore, a great desideratum. Such a combined physiological laboratory and growing room has recently been constructed at Beloit College and the following account of it with the above cut is given in the hope that it may suggest a similar possibility in some of our high schools.

The Beloit laboratory is over fifty feet long by twenty-five wide and was constructed from an unused portion of the fourth story of our science hall. The remodeling consisted largely in the changing of partitions and in placing skylight in the south and west roof spaces between the windows of the original rooms. The room thus made available for plant growth and experimentation was then furnished, as shown in the cut, with wall growing tables and a large chemical work table running lengthwise of the room. This work table is supplied with water and gas for each student. A somewhat novel feature of the new laboratory was introduced by covering the floor with "Inderoid Roofing," sold by the Interior Wood Work Company of Milwaukee. This roofing has proved durable and serviceable, since it enables us to spill as much water as we please without danger of leakage into the laboratories below. This feature also allows of frequent sprinkling of the floors with the hose to keep the room moist enough for plant growth. Another feature of some interest is a series of soil bins along one side of the laboratory which contain clay, sand, humus, and other soil types, with a potting table above the bins for convenience in potting plants and sifting the soil without littering the main laboratory floor. Adjacent to the laboratory is a convenient store room and in a south window a growing case for plants needing more moisture than can be maintained in the main laboratory. A dark chamber and an aquarium complete the essential equipment of the laboratory, which is already indispensable to the entire department, not only for plant physiology proper, but as a general growing room for supplies of fresh plants for Morphology, General Biology, and Anatomy.

Last semester we spent considerable time in experimenting upon plants which could be grown easily and which were likewise good material for experimentation. Among the plants experimented upon, Indian corn proved to be one of the easiest growers and one of the most serviceable for much experimental work on photosynthesis, absorption, and root pressure. For color screen work in photosynthesis and for starch translocation and rapid formation corn was found to be particularly favorable material. It proved, also, to be the best plant tried for root pressure. This year I hope to have my students work out as completely as time will permit the physiology of the corn plant, using the suggestions which came from last year's experience.

The total cost of the laboratory was about eight hundred

dollars, but a smaller room with many of its essential features could be fitted up at a comparatively small cost. It is hardly necessary to add that no really effective work can be done in plant physiology without a room in which plants can live a normal life. The appeal for a suitable equipment for teaching plant physiology in the high school comes from many sources. Agriculture, as it is being taught in many high schools, is on the plant side too often studied without the necessary physiological basis. Forestry, horticulture, and plant breeding combine physiology, morphology, and ecology in such a way as to make physiology necessary for their proper appreciation. My own conviction is that the botany in high schools should be practical and applicable to what the student will see and do in after life. It must never be forgotten, however, that the basis of all practical scientific work with plants is found in the knowledge of pure science and in skill gained by real science work. It will never do to study about agriculture and forestry without understanding the scientific principles which underlie them. We need to equip our high schools for good work in physiology and morphology and use as materials for study and scientific training plants which are familiar, useful, and ornamental.

REPORT OF THE COMMITTEE ON THE EXPERIMENTAL INVESTIGATION OF THE TEACHING OF BIOLOGY.[1]

Your committee finds itself obliged to report that it has not been able to uncover much research of an experimental sort upon the teaching of biology, other than that which enters into the program of this meeting. We have not, therefore, been able to perform that part of our duties which relates to coöperation with members engaged in research. We have, instead, given our attention to the preparation of a report upon the importance and opportunities of such work. We believe that it is necessary to urge this upon members.

The present is a critical period in the history of science instruction. Botany and zoölogy have both been in the curriculum long enough to demonstrate what they can do, and we are already hearing it said that they have not fulfilled their early promises. We are going to be called upon to demonstrate our right to the time we occupy in the course of study, to show

[1]Presented before the Biology Section of the Central Association in Cleveland, Ohio, Nov. 26, 1910.

exactly what contribution we make to the pupils' mental furniture, and to lay down definitely the broad principles upon which we stand. While we all feel confident of the correctness of our position we are not fortified with arguments founded upon definite data; we are obliged to rely upon impressions and claims. If we are to maintain the place that belongs to us we must have *facts*, indisputable facts.

These data can be secured by no one else than the teacher of classes in biology. He alone has before him the materials for experiment and observation—the pupils. The future position of biology therefore depends largely upon the present activity of teachers in this branch of investigation.

If conceived and carried out in the true scientific spirit such work deserves to stand by the side of any other scientific research. He who ascertains how the sum of human knowledge may be efficiently used to further the development of the race is indeed deserving of more honor than he who only adds a few more facts to an already overwhelming total. The teaching biologist has the same standing as the investigator in pure science, just so far as he becomes a creator of new knowledge or learns to use the old rationally rather than empirically. We must cease to work by "rule of thumb" and determine the laws of our profession.

It is true that research upon the pedagogical side of biology has not been prominent at our universities or in our scientific periodicals, but this has been due to the inactivity of those who are in the field rather than to any lack of appreciation. The universities are not unreceptive to the idea; several of the greatest educational institutions within the territory of this Association are ready to grant higher degrees upon theses such as should result from a scientific study of some of the problems that arise in connection with instruction in biology. It ought to appeal very strongly to teachers that it is now possible to secure research material from the routine work of their classes and at the same time to inject into the class work the added interest which comes from a new point of view on the part of the teacher.

The difficulties of carrying on research in pure science and at the same time keeping up teaching are well set forth in the last presidential address before the Botanical Society of America, an address that ought to be read by every member of this section. (Printed in *Science*, N. S., XXXI, pp. 321-334, March 4, 1910.) Particularly appropriate are those sections in which

the author insists upon teaching in itself as a scientific contribution. The recognition of this fact will give many of us an entirely new outlook.

Such investigation does not of necessity imply any disturbance of the class routine—it does mean keeping accurate records and the analysis of these records. Most of us consider each class we teach as an experiment by means of which we shall learn to teach better the next time the course is repeated. Few of us subject our records, quantitative, or descriptive, to such analysis as would enable us to emerge at the end of the year with anything more definite than "impressions" as to the results of the course. We must learn to follow our own experiments to the end, and to put our conclusions and data in such form that they are useful to others.

As an aid to stimulating interest the committee adds a somewhat random list of subjects which seem suitable for investigation in the class room.

Segregation of the sexes in biology classes.

The value of voluntary field work.

Alternative courses—morphological, ecological, economic, etc.

Periodicity in a child's interest in biology—is there a stage in maturity at which interest in biology is at a climax?

The lines of pupils' interests.

The lines of pupils' needs.

Practicability, limits, and materials of instruction in sex hygiene.

The proper place of demonstration, group experiment, and individual experiment.

The use of economic materials for illustration.

The exclusive use of economic materials.

The proper relation of text-book and laboratory.

General biology vs. separate courses.

Botany and zoölogy in a general science course.

What scientific facts and principles are prerequisite to courses in botany and zoölogy?

Just how practical are Department of Agriculture bulletins in the hands of pupils?

Form and content of laboratory notebook.

The place of drawing in elementary instruction.

Laboratory manuals and their proper place.

The relation between training in art and the drawing work of the laboratory.

Overcoming distaste for necessary morphological work.

Relation between field work and laboratory study: which should precede?

Outdoor laboratories: what should they contain and how be used?

Advantage of laboratory study by topics vs. types.

Value and method of the use of the library in courses in botany and zoölogy.

What books should be placed in such library?

Respectfully submitted,

W. L. Eikenberry, *Chairman*,
University High School, University of Chicago.
T. L. Hankinson,
Eastern Illinois Normal School, Charleston, Ill.
H. C. Drayer,
Yeatman High School, St. Louis.
F. C. Lucas,
Englewood High School, Chicago.
W. W. Whitney,
Bowen High School, Chicago.

Committee.

At the Cleveland meeting the above report was adopted as the expression of the Section. The committee was continued with instructions to assist in every way possible the development of experimental research in the pedagogy of biology. The committee hopes that the members of the section will coöperate actively in this matter. It is essential that every member who is carrying on such experimental investigation, or who knows of someone who is doing so, shall inform the committee. The definite solution of simple problems is as welcome as the elaborate treatment of more complex matters.

The committee has also been directed by the Section to render available so far as practicable the literature of the subject. This will be accomplished in part by publishing a bibliography in School Science and Mathematics. It is planned to include titles not only in the biological field, but also in other sciences when the method seems especially applicable to biology. Some of the more important titles in the scientific study of education in general will also be cited. It is hoped to publish the first section in the next number, and other titles will follow as they are prepared.

W. L. Eikenberry, Chairman.

PRACTICAL WORK IN AGRONOMY.[1]

By Willard N. Clute,
Curtis High School, Chicago.

Two seasons' work with classes in agronomy have resulted in the following conclusions:

The educational value of agronomy is as great or greater than any other phase of botany. It is a training of both mind and hand, makes the pupil a more useful and contented citizen and furnishes an outlet for the motor activities of the child.

Agronomy should be part of the regular work of the school with the same standing as the other sciences. The gardening work should be conducted during school hours and full credit given for it as in any other study.

It should not be taught as a recitation study, and if the gardening space cannot be secured the course would better be omitted. Practical work in cultivating plants should be an integral part of the course.

Agronomy cannot be profitably taught unless preceded by at least a half year devoted to the structure and physiology of plants. If a year can be given to agronomy, the first half of the course should be devoted to structure and function. In any event, the course should be so arranged that the part directly concerned with agronomy comes in the spring. If a half year devoted to the spore plants is also given, this may follow agronomy or immediately precede it, the former by preference.

In teaching the subject, a distinction must be made between the schools of city and country. The city school should not attempt to teach the care of stock, care of milk, or the growing of cereals, but it can teach well such subjects as growing vegetables, care of lawns, planting of shrubbery, grafting, budding, the propagation and amelioration of plants, the methods of combating insect and plant pests, etc., etc.

The garden should be safe from marauders and if possible located near the school, but if within fifteen minutes' walk the course can be given. The nearest vacant lot can be used, though since a degree of permanence in much of the planting is highly desirable, the garden should be owned by the school if possible. If ground must be purchased it will usually be found to cost

[1]Abstract of paper read before Biology Section at Cleveland meeting, Central Association Science and Mathematics Teachers, Nov. 26, 1910.

less than to fit up a laboratory and the expense should not stand in the way of the course.

The garden should have a grass plot and a border of hardy shrubs and perennials to familiarize the pupil with the ordinary plants used in such planting. There should also be plots of certain permanent vegetables such as rhubarb, asparagus, artichoke, sea kale, and the like.

Each pupil should have a garden plot of his own and be allowed to carry away the crops produced. Gardening in partnership should be restricted to various experimental plots in which can be grown the plants less common in gardens, such as hemp, flax, alfalfa, tobacco, sweet potatoes, okra, chufas, etc. All sorts of variations from the normal may also be collected here—four-leaved clovers, albinos, yellow berried forms of plants that have normally red berries, etc.

Among the best plants for producing crops before school closes are lettuce, spinach, cress, onion sets, and radishes in variety. Plants that may be left to take care of themselves until school resumes in autumn are parsnip, oyster plant, carrot, beet, and potatoes. Good crops to follow the early crops which, however, require attention in summer, are sweet corn, cabbage, celery, turnips, and winter radish. Tomatoes, beans, and kohlrabi may be planted as late crops or used with the early crops.

The garden is invaluable for growing illustrative material for the beginning and other classes and as a place in which to study systematic botany. With a properly planted garden systematic botany classes may see each flower identified as it grows without losing time in searching the surrounding country for it. Beginning classes in botany may also do much of their work in such a garden.

Systematic botany with a good manual is strongly recommended for utilizing the time not needed for cultivating the garden. The theoretical work in both agronomy and systematic botany can be given in school before it is possible to begin work out of doors and the practical work of both studies taken up as the weather permits.

A list of the subjects treated in the course given by the writer may be had upon application to him.

VARIATION AS A TOPIC IN HIGH SCHOOL BOTANY.

By W. L. Eikenberry,
University High School, University of Chicago.

The subject of variation has been in high school botany as long as the theory of evolution has been a subject of instruction. It is usually, however, relegated to a very minor place in connection with the discussion of evolution somewhere near the close of the course. It is practically never presented in connection with any experimental or observational data which have been a part of the pupil's experience. The pupil commonly studies but a single individual of each type, or but a few individuals, with nothing to call his attention to such variations as exist. Then, after he has been thoroughly grounded in the idea of the fixity of types, he is called upon to base an explanation of the genetic relations of these types upon a variation which he has not observed in the course of his work. It is really surprising that the average pupil succeeds so well in executing the mental somersault which is necessitated and accepts the *fact* of variation though he may not appreciate its meaning.

In many late texts variation is given a more prominent part but always in association with plant breeding rather than in preparation for an understanding of morphology and evolution. It is the opinion of the writer that the subject does not present any difficulties which debar it from the early part of the work, and that it is of the greatest advantage as an introduction to the morphological study. If it is placed at this point in the course it enables the evolutionary viewpoint to be maintained throughout.

It may be worth while to suggest, in a concrete way, how variation may be presented to a high school class. This year two varieties of wheat were selected for comparison, not because there was any reason to suppose that they were particularly favorable, but merely because the material happened to be at hand in the school museum. It was decided to investigate the variation of the number of grains in a head.

The two varieties of wheat differed considerably in general appearance. One was smooth with unbranched head (A) while the other was bearded and the head was branched (B). Both appear to have been grown by "dry farming" in Montana.

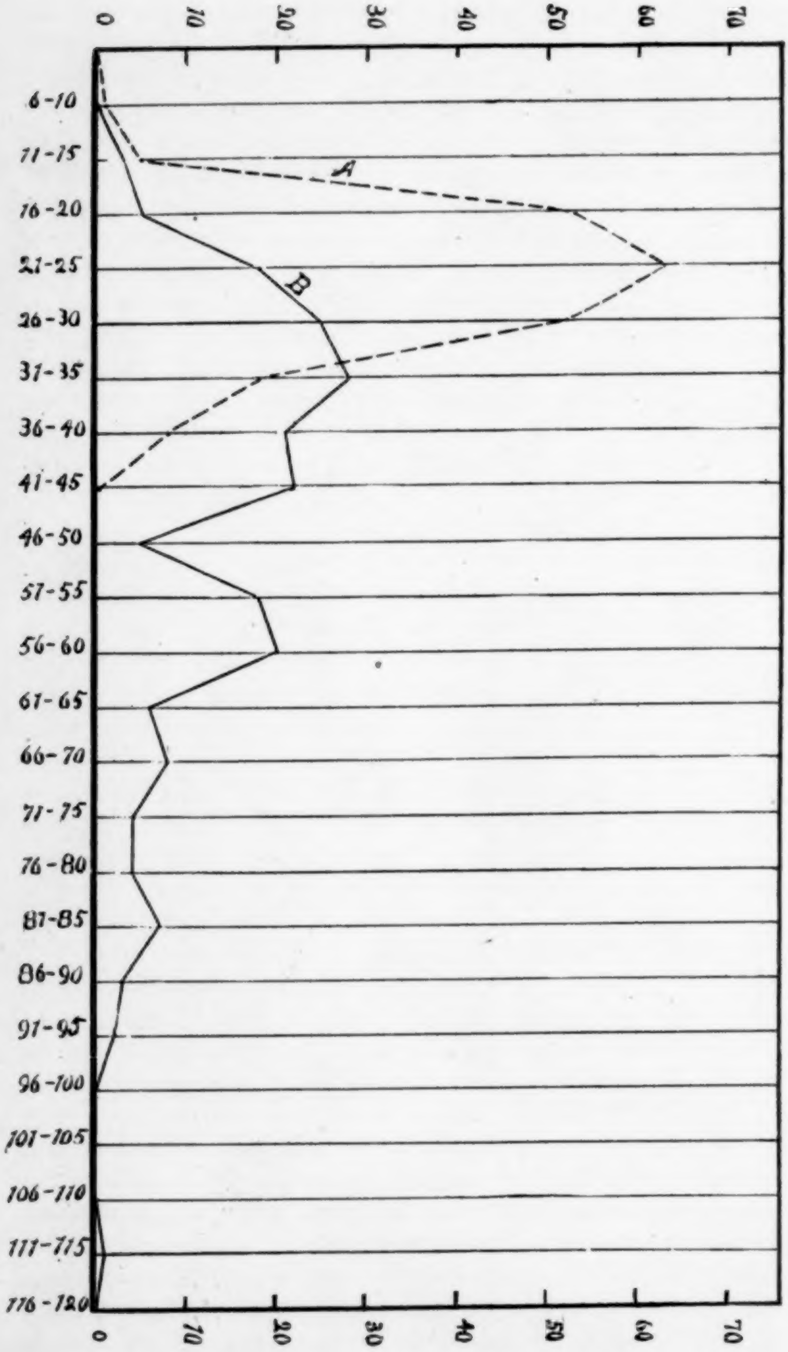
The class was divided into two sections and each section counted the grains in a hundred heads of one variety. Each

pupil counted about eight heads. The combined results were expressed in the form of a frequency table similar to that reproduced here, and later put in the form of a graph. Each pupil reproduced the frequency table in his notes and constructed his own graph from it.

The work was repeated by one of the classes of the College of Education which had become interested in the same subject and the subjoined table and graph is made up from the work of both classes and therefore shows results with two hundred heads of each sort—the results with one hundred heads were not materially different. All the main features of the curves are found in each set of results. It would appear, therefore, that results which are valid for demonstration may be secured from so few as one hundred countings.

No. of Grains	No. of Heads	
	A	B
6 - 10	1	
11 - 15	5	3
16 - 20	52	5
21 - 25	63	18
26 - 30	52	25
31 - 35	19	28
36 - 40	8	21
41 - 45		22
46 - 50		5
51 - 55		18
56 - 60		20
61 - 65		6
66 - 70		8
71 - 75		4
76 - 80		4
81 - 85		7
86 - 90		3
91 - 95		2
96 - 100		
101 - 105		
106 - 110		
111 - 115		1

The class found no difficulty in discovering the main facts it was desired to impress, namely; the existence of variation, its arrangement about a central or maximum point, and its different distribution and limits in different plants. Incidentally, the peculiar form of the curve of the bearded wheat (B) gave rise to some interesting discussion regarding plant varieties, plant breeding, and the probable excellence of the two varieties of wheat in


0
10
20
30
40
50
60
70
6-10
11-15
16-20
21-25
26-30
31-35
36-40
41-45
46-50
51-55
56-60
61-65
66-70
71-75
76-80
81-85
86-90
91-95
96-100
101-105
106-110
111-115
116-120
A
B

cultivation. Examination of the curve will suggest interesting problems to anyone who is interested.

It is possible that breeding experiments may be instituted on the school grounds next year, selecting for planting such heads as have a number of grains corresponding to one of the several maxima in variety B—eighty-one to eighty-five grains for instance. Such experiments as these could not fail to arouse interest among the pupils of succeeding years.

The frequency table and curves are appended. As noted above, these are the combined results including two hundred heads of each variety.

HIGH SCHOOL OR COLLEGE BOTANY, WHICH?

By Willard N. Clute,
Curtis High School, Chicago.

Those most interested in the presentation of botany in the high school are often heard to complain that the subject is not taught as well as it might be, but it is seldom that any attempt is made to give a reason for the ineffectual way in which it is handled. We realize that something is wrong without being able to suggest a cure. Possibly several factors influence the result —among which we may at once name poorly equipped laboratories, inadequate material, unsatisfactory text-books, and inefficient teachers—but it seems to me that the most important factor is found in the way in which we confuse the botany of the high school with that of the college and university.

At first glance it may seem absurd to insist that there are two phases of the subject and that one is more appropriate for presenting in the high school than the other, but a little thought must show, since the aims of college and high school are somewhat different and the attitude of their respective students decidedly so, that if there are not two phases there certainly ought to be. The college student is primarily after knowledge and wishes to get it by the most direct method possible. His teacher, therefore, does much of the reciting, though we commonly call the recitations lectures, and illustrates his remarks by specimens, photographs, drawings, and lantern slides. Such a course is, in fact, largely a distribution of information. The instructor attempts to cover completely any subject upon which he may touch, assigns much outside reading, exacts a careful study of the text-

book in the laboratory, and requires the notebook to be a repository of facts gleaned by the student.

The high school teacher, on the other hand, is not especially concerned in developing botanists. With him the first effort is to inculcate the scientific attitude and to make his pupil a seeing, investigating, and reasoning being—a credit to his teacher and a joy to the college that may later attract him. Nor is the raw material out of which such paragons are to be made very strongly inclined toward botanical knowledge. It consists of students who will learn things botanical if they must, but who much prefer to play. It is rare that any large number elect botany because of a love for the study. The prospective field trips allure some, others take the study because a chum does so, or because it looks easy, or because a credit in science is needed for graduation. Such students may be drilled into exhibiting no signs of restlessness under the college method of presenting botany, but they rarely work up much interest or enthusiasm for it.

It is no kindness to the high school pupil to recite his botanical lessons for him, no matter how delightfully illustrated they may be by lantern and microscope. Indeed, such methods may be termed the worst possible, since it takes from the pupil the joy of discovery and thereby lessens that interest in the study upon which every good teacher rightly depends for carrying the class successfully through the course. What the high school student needs is not to be told about the subject, but to be given a set of fairly searching questions and proper materials and directed to get his answers for himself. A laboratory manual that tells the pupil what he is to see, is about as much out of date as the old botanical course that consisted mainly in "analyzing" flowers. And the text-book should be rigidly excluded from the laboratory for the same reason that the college professor should be—because it tells too much. We wish the student to find out for himself. For be it remembered that the use of botany in the high school is primarily to give him power, rather than to fill him up with information.

It is not essential that every phase of botany touched upon in the high school course be exhaustively studied and little assigned reading is necessary. The time used in reading about things may be more profitably used in studying the things themselves. Nor is the notebook here a mere storehouse of facts. It should be a carefully written account of what the pupil has worked out for himself.

Poor teaching has spoiled more good students than lack of knowledge. The college-made instructor, whose claim to be able to teach botany rests mainly upon the fact that he has taken a botanical course, often adds to the confusion of the subject by attempting to give the pupils entrusted to him a dilute and weak imitation of the college course, forgetting altogether to consider their special needs. If botany is ever to take its rightful place in the list of high school studies, there ought to be a place somewhere where the college-fed person may be taught how to teach. Anybody can ask questions that require only the knowledge gleaned from the books to answer, but it takes little short of genius to ask questions that shall drill the pupil in expressing the results of his own thought or indeed to make him think at all.

The college-bred teacher is likely to be further hampered by a text-book written by a college man with no high school experience and no more idea of what high school pupils require than a Hottentot. The very language used needs an interpreter. The substance of the course may not be over the heads of the pupils, but the language certainly is. The writers of books instinctively fall into the habit of using words derived from the Latin and signally fail to perceive that the child's vocabulary consists largely of shorter words derived from the Anglo-Saxon. As to the substance of the course, it is probable that we shall never have an entirely satisfactory book until college man and high school man collaborate in producing it.

Without doubt the college method of presenting botany is best for the college, but it is here contended that it is not desirable for the high school because of the different results aimed at. If it be concluded that the first function of the teacher in the high school is the furnishing of information, then the college method is probably the better, but otherwise it is not, since the high school usually aims at giving power as well as knowledge through this study. Teachers who have difficulty in making botany attractive to their students, may find that the trouble lies entirely in this failure to distinguish between two phases of the subject. Under such conditions, the other phase ought to be given a trial at least, keeping in mind the viewpoint of the child and the things that interest him.

THE LOCUS PROBLEM IN GEOMETRY WITH SOME DISCUSSION OF THE UTILITIES IN GEOMETRIC STUDY.[1]

By Fletcher Durrell,
Lawrenceville, N. J.

According to my experience and observation the most profitable feature of meetings like these is the interchange by teachers of actual class room experience. It is an advantage to know the methods of fellow teachers, whether we see fit to follow them or not. Let me then begin the present discussion by a statement of how I treat the matter in hand in the class room, and follow this statement by some general observations on the larger bearings of the question under consideration.

The term "locus," when first mentioned, is apt to repel the pupil by its strangeness and abstractness. In the space of a few weeks after beginning the study of geometry the pupil is introduced to a large number of new terms and ideas, and at the mention of this additional term with a stranger name than any that precedes it the pupil is apt to get discouraged and want to quit. Hence it is my habit to introduce the topic in hand with considerable explanation, using the word "path" instead of "locus." Thus I say, "If a point moves so as always to be three inches from a given fixed point, what will its path be or in what path will it move?" So of a point moving so as to be always two inches from a given straight line, etc.

On the other hand the loci naturally arising in Book I are so simple and obvious as not to justify in the pupil's mind the effort necessary to master this new concept; hence it seems desirable at the outset to introduce broader illustrations than those supplied by Book I. Thus we can make a natural transition to the conic sections by asking first what will be the path of a point moving so as to be equidistant from two points; or then from two lines; and then from a point and a line, thus introducing the parabola. Boys of course are always much interested in learning that a projectile, as the baseball every time it is thrown, or the football every time it is kicked, moves approximately in the arc of a parabola, a curve determined in the simple way just mentioned. Similarly I call the attention of classes to the ellipse as the path of a point the sum of whose distances from two fixed points is the same and hence being

[1]Read before the Philadelphia Section of the Association of Teachers of Mathematics the Middle States and Maryland, May, 1910.

closely related to the circle; to the fact that the orbits of the earth and other planets are ellipses, the sun being at a focus; and during the present year to the orbit of Halley's comet. I also mention the fact that I once had occasion to lay out an eighth of a mile elliptical running track for boys and did it by simply driving two stakes into the ground, fastening to them the ends of a piece of string of proper length and locating various points on the track by stretching the string tight in various positions.

I think that for the sake of completeness I should mention another somewhat informal illustration that I sometimes use. Let PQ be a ladder leaning against a perpendicular wall BP, and R a boy sitting on a rung at the middle of the ladder. Suppose another boy to take hold of the ladder at Q and pull the end of it out along the ground. What will be the geometric path in which the boy at R will move? Boys are always greatly surprised and interested to learn that the boy at R moves in the arc of a circle. This illustration also enlists the interest of the class by appealing to the spirit of mischief which lurks in every boy. In trying to arouse interest we thus wander among the stars and then come down to earth. I may say in passing that the keen interest aroused by this illustration has other advantages; the interest aroused is strong enough to carry us through the statement and appreciation of two other somewhat important new ideas. The first of these is the idea of the constant. For on constructing a circumference on PQ as a diameter this semicircumference is seen to pass through B and hence the line $RB = \frac{1}{2}PQ$ in all positions, or is a constant. Also we introduce auxiliary quantity in the somewhat complex form of a series of semicircles on a variously placed diameter.

So much for the method of introducing the idea of a locus to a class. Now let us take up the matter of the solution of original exercises calling for the discovery and proof of a specified locus. The standard methods of discovering a locus are either to locate a number of points each satisfying the given conditions and observe the relation of the points constructed (the observed relation to be tested by demonstration later), or to reduce a required locus to one already known. It is to be noted that the first of these opens the way to the more scientific definition of a locus as the aggregate of all points satisfying given geometric conditions. These matters, however, are not of sufficient difficulty or individuality apart from the treatment

of originals in general to call for extended discussion at this place. The specific difficulties in treating this topic arise in demonstrating a locus. The time at our disposal in teaching the subject is so short, the locus is often so evident, while the complete demonstration, involving as it does both a positive and a negative part, is often so long and uninteresting because unproductive of anything different from what our intuitions give, that something must be done to put the matter in really practical class room shape.

Acting on this theory or principle then my primary object in certain places in geometry is to keep up interest and achieve useful results, while at the same time I achieve such domains of demonstration as I can or have time for. In all cases, of course, where the domain of logical demonstration is limited it is of the first importance, in order to prevent loose work, in each case to have it clearly understood what demonstration or what parts of a demonstration are omitted. Thus in teaching construction problems, I frequently assign a lesson thus: Page 169, Ex. 2, Ex. 4 (lines only), Ex. 6 (lines only), Ex. 7 (lines only). "Lines only" means that the statement and proof are to be omitted, but that the diagram is to be made and left complete in every detail. By having only one problem in four or five written out in full, the pupil is kept busy at what he most likes, is stimulated by the sense of progress, and is able to cover fifty per cent more ground than he otherwise would in a given time.

The relations of different parts of the work are also by this means thrown in to a new and sharp perspective. Similarly in teaching numerical applications of geometry time is often saved by assigning a lesson thus: Page 307, Ex. 5 (ans. to 3 decimal places), Exs. 6 and 7 (ans. in radicals). The pupil thus first masters the unit process and then gives it a rapid multiplier.

It is of the first importance that the description of the locus made by the pupil be accurate and complete. Thus for instance the pupil will often limit his description of a locus to the statement that the giveh locus is the circumference of a circle. In such cases of course the pupil should be required also to specify the center and radius of the circumference. When the actual demonstration of a locus is taken up, the matter is complicated by the fact that a complete proof is double or involves the proof of two closely related theorems. Hence certain pitfalls as well as tedious semi-repetitions arise.

As preliminary steps in learning the process of the complete

demonstration of a locus it is often well to assign the different parts of such a demonstration as separate exercises. Thus a pupil may be asked to prove that every point satisfying given conditions lies in a specified circumference; and, as a separate exercise, to prove that every point not satisfying the given conditions does not lie on a specified circumference.

Let us now take up the broader question of the educational significance of the locus. It is a familiar statement that two classes of utilities, uses, or values are connected with any subject of study, viz: 1. Technical values; 2. Cultural values. Yet on examination this distinction as it is usually understood is found to be a somewhat loose and superficial one. It is my theory and belief that when the so-called cultural values, or broad and abstract utilities as we might call them, are fully analyzed and precisely and fully understood, they will be found to underlie technical utilities and to include them as particular cases or details; and that as a consequence in all our work these broad and abstract utilities should be made primary. Whereas technical utilities are used frequently by only a few persons, the so-called cultural utilities are general and pervasive and used every day by almost all, consciously or unconsciously for technical as well as cultural purposes.

I may say that I have lately collected for class room use several hundred applications of geometry to mechanics, engineering, physics, astronomy, architecture, and ornamental designing, etc. Of course I have tried to make these applications as interesting and useful as possible; yet after all I regard them as only of secondary importance in the study of geometry, even for students preparing for engineering courses.

Thus when I spoke in this place a year ago I tried to show among other things that while the principles of geometry will be rarely used by any of us in such a technical application as that of finding the height of a steeple or tree by means of shadows and similar triangles, yet if we acquire in the study of geometry the mastery of the use of auxiliary quantity in the simple, systematic, penetrating form there possible, such mastery will be of service to the physicist every day, to the chemist, engineer, merchant, day laborer, cook, housekeeper, entertainer, every one of us many times every day. Or to look at the matter more specifically, it is a matter of common note that the greatest men have been the greatest masters of auxiliary quantities and entities; thus we have the proverb that time and tide are

on the side of the great man; similarly Newton said that he had achieved results because he stood on the shoulders of giants. It is also a matter of note that the greatest men, men like Helmholtz and Lord Rayleigh in physics, have been geniuses in getting results out of simple near-by objects like pins, strings, and sticks, that is, have been the greatest masters of natural near-by auxiliary quantity.

Here we have a general, or abstract, or so-called disciplinary or cultural utility acting in a thousand concrete ways to produce concrete results, as well as acting to produce purely cultural results. The principle is so important that I will illustrate it further before applying it to the matter of the locus. Take for instance the utilities in the study of algebra and note how the disciplinary or cultural values underlie and include the technical ones and indeed may be made to pervade all life and add economy and efficiency to its processes. We would agree that an example of the technical utility in algebra would be the use of algebraic equations to determine the number of pounds of coffee of different grades and price to be mixed in order to produce another grade of a given price; or of an equation to represent laws of falling bodies, or orbit of a planet around the sun. Yet in our individual lives who of us has had occasion to make any one of these applications of algebra? On the other hand if we understand the inner meaning of symbolism we can make daily use of the powers gained by the study of algebra and often in very important ways.

In teaching certain propositions in geometry, such for instance as the theorem that in the same circle or in equal circles equal arcs are subtended by equal chords, I have often noticed that even though the pupil at the outset of the demonstration had a clear grasp of what was given equal, the arc, or the chords, as the demonstration progressed this matter slipped away from him and for this reason the whole proof became confused. It occurred to me that this difficulty might be obviated by having the pupil while stating the proposition put small equality marks on the parts given as equal: thus when the arcs are given equal, equality marks are to be placed on the arcs. This device proved a complete success. It applies to other somewhat different and more difficult propositions. Thus it is desired to keep in mind that one chord is given greater than another in a circle, the letters *g* and *l* may be annexed to these chords. This is an application of the spirit of algebra rather than its technique, and

hence is an illustration of how algebra may be made of daily help to the teacher. Or again if I have three or four sets of papers which I must handle together yet wish to keep distinct, I can keep them distinct by placing them upon each other at different angles; thus using difference in position as a symbol of difference of quality or relation. This seems a small matter yet note to what the principle involved may lead. In writing numbers it was once necessary to write, for instance, 7th8h5t3u. But after the symbolic significance of relative position was recognized the above number could be written 7,853 with the illimitable economies and advantages which follow in treating number by the Arabic notation. Similarly hosts of other uses, conscious or unconscious, often personal to the user and temporary, flow from a grasp of the inherent principles of algebra. All ordinary written and spoken language is a form of algebra, and a full grasp of the cultural values of algebra, or what I would prefer to call its abstract yet everyday utilities, should add new economy and efficiency to our use of ordinary language, as well as to many other processes of life.

For us as teachers and to our pupils as students of geometry applications of the locus principles are the uses of loci as aids in solving other exercises, as in constructing a circle of given radius which shall pass through a given point and touch a given line. Later uses of course come to some pupils in the study of analytical geometry and calculus. A broader and more pervasive application of the spirit or essence of the locus principle is the following: If we take an idea (or term) like "white," and another like "man," and cause them to intersect we get a new concept, viz: white man. By reuse of the two known terms we are saved the labor of inventing a new term as Caucasian. Similarly a chemist combines or causes two cheap substances like iron and carbon to intersect or meet and gets a diamond. So in life in general by causing an endless variety of objects and ideas to intersect we obtain new results full of economic and efficient properties. The study and grasp of loci, rightly carried out, consciously or unconsciously should aid in perfecting this element of life.

In conclusion allow me to say a word on the relation of the general question just raised to present general educational tendencies. In the American public at the present time there is a strong and growing demand that education be made more practical. Not seeing another kind of definite practicality in sight,

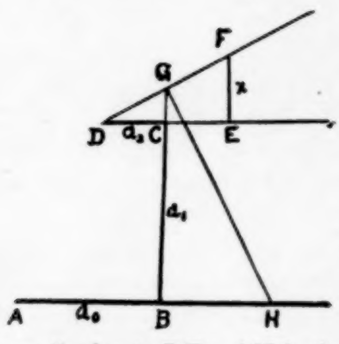
many people are demanding that schools be vocationized. The result is a danger of a swing into very costly materialistic experiments. I believe that the root of the trouble lies in the fact that concrete utilities have been made clear to the people, while comprehensive inclusive utilities, which both prepare for life, and also in the most fundamental and effective way for direct breadwinning activities have not been analyzed and made definite. For instance we do not have a sufficiently definite grasp of the broader utility elements in education to enable us to impart more to one pupil, and less to another, according to the needs of a given case. This more definite working out of the fundamental utilities in different educational activities impresses me as the most important educational problem now awaiting solution. If we can show that mathematics may be made a definite means of mastering the use of auxiliary quantities or objects, of perceiving uniformities and diversities among all objects whatever and hence of grouping them systematically, or reusing what knowledge and power we possess in new conjunctions, of devising units and multipliers in every department of knowledge and activity, of correcting or preventing errors, of solving paradoxes, and if we can show that mathematics is especially fitted to perform these services because of the relative simplicity of its concepts, its relative freedom from entangling alliances, and because of its penetrating power, we shall not only have made clear to ourselves that the future of mathematics or something closely resembling it as an educational discipline is secure, but shall also have done something to solve the general problem just mentioned.

A GRAPHICAL SOLUTION OF THE QUADRATIC EQUATION.

By Albertus Darnell,
Detroit, Mich.

In a short series of lectures given in February of this year at the University of Michigan Professor Carl Runge of Göttingen, among other applications of graphical methods, mentioned a very interesting solution of the quadratic equation. Professor Runge, it will be remembered, was Kaiser Wilhelm Exchange Professor at Columbia University during the past year.

First consider the following graphical calculation for determining a line which shall represent the value of $f(x) = a_0+a_1x+a_2x^2$ for some particular value of x.



Lay off AB $= a_0$, BC perpendicular to AB and equal to a_1, CD perpendicular to BC and equal to a_2. Now make DE equal to the unit of measure, EF perpendicular to DE and equal to the value of x for which we wish to compute the value of $f(x)$. Draw GH perpendicular to DF. AH is the linear representation of $f(x)$ when x equals EF.

It is obvious that the triangles of the figure are similar. Therefore we have

x : CG = DE : a_2.

Remembering that DE is the unit, this gives

CG $= a_2x$, and from this we have

BG $= a_1+a_2x$.

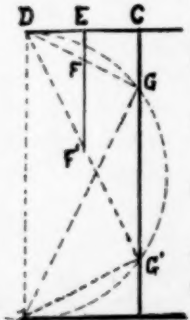
BG : DE = BH : x.

Making the proper substitutions, this gives

BH $= a_1x+a_2x^2$, and from this we have

AH $= a_0+a_1x+a_2x^2 = f(x)$.

If the value of x had been so chosen that the point H had fallen at A we should evidently have had a value of x satisfying the equation $f(x) = 0$. This reduces the solution of the quadratic to the problem of constructing a right triangle on AD as hypotenuse with the vertex of the right angle on BC.



The following figure shows the application of this method to the equation $2+5x+2x^2 = 0$.

EF and EF' are respectively the roots $-\frac{1}{2}$ and -2, DE being the unit.

The variations of the figure resulting from changes in the signs of the coefficients present no difficulty.

An account of the literature on the subject can be found in Encyklopädie der Mathematischen Wissenschaften.

THE PRESENTATION OF POSITIVE AND NEGATIVE NUMBERS.

By Merton T. Goodrich,
Dixfield High School, Dixfield, Me.

The beginners' class in algebra is reciting. The teacher has just finished the discussion of the assigned work, some elementary problems in simple equations. He pauses an instant, then, without speaking, turns and walks to the corner of the room. He takes a thermometer from the hook where it has been hanging and faces the class. Every eye is upon him. That moment of silent acting has attracted and fixed the attention of the pupils.

The teacher asks them to read the temperature and writes on the board their complete answer, "65° above zero." He asks if the temperature ever goes below zero. If the school is in the northern part of the United States, the pupils will promptly answer that it does. "How cold have you known it to be?" the teacher asks of someone. The answer, "25° below zero," is written on the board beneath the first expression. The teacher then asks the class if they think there may be a shorter way of writing these expressions, calling attention to the fact that the chief work of algebra is simplification and that algebra uses symbols to represent words. The majority of the class believe there may be a shorter way. All are interested to see what the way may be. The teacher points out that the sixty-five degrees is above zero, is more than zero, and that the degrees have been added one above the other until there are sixty-five of them. He emphasizes the idea of addition and then suggests since the plus sign means addition that it be placed before the sixty-five to show that the sixty-five has been added to zero. While he is speaking he writes, "+65°," opposite the corresponding words. In an instant the pupils decide for themselves that this is reasonable and right. The teacher then asks the class, "Now, how shall we write twenty degrees below zero in a shorter way?" Someone answers, "Minus twenty degrees." This actually happens before the word negative has been mentioned. When the teacher writes, "—20°" after the corresponding words, he points out that the symbols and the words mean the same thing and are simply different ways of expressing the same idea. He emphasizes the fact that minus means below zero, less than zero

that zero is a point on the thermometer from which we can count the degrees.

The teacher then makes it clear that plus and minus numbers do not always refer to temperature. He asks the class how far it is from the home village to a neighboring one that is up the river. He writes their answer, "5 miles up from ——." He asks the class to name a place the same distance down the river, and writes the distance beneath the first, "5 miles down from ——." To the teacher's request for a shorter way to write these expressions, someone gives the answer, "Plus five and minus five." The teacher then asks, "Where is the zero point in this problem?" After a little hesitation, someone names the home village.

The pupils, who are making these replies, have never studied algebra before. They are not only being taught algebra, but they are being taught to think. They are associating the idea of plus and minus numbers with the familiar geography of their town and its vicinity. Now, and only now, is the time to draw the well-known diagram of a straight line. But it means more to the pupils than a straight line with a lot of figures fastened to it by plus and minus signs. It is a diagram of the distances between familiar places.

Using the numbers, plus five and minus five, the teacher shows the important use of plus and minus signs to distinguish between such numbers. Turning back to the problems in temperature, he shows how the signs distinguish between such temperatures as ten degrees above zero and ten degrees below zero.

He may now employ the time-worn but still valuable problems in debt and wealth to further illustrate the idea. It is usually necessary to take time to show the boy who owes fifty cents and has no money in his pocket that he is actually worth less than nothing. This is most easily done by comparing his financial standing with that of a classmate who neither has anything nor owes anything. He then sees that he is worth fifty cents less than his classmate who is worth nothing, and that his wealth may be represented by minus fifty.

This discussion opens the way to the explanation of the addition and subtraction of plus and minus numbers. The teacher asks the boy who is in debt fifty cents, how much he will be worth if someone gives him a twenty-five cent piece. He gives the correct answer. The teacher then asks him how much he will be worth if someone else give him a fifty cent piece. If he

hesitates, some other member of the class gives the correct reply. Similar exercises are used to fix the idea in the minds of other pupils. Two or three pupils must not be allowed to do all the thinking. The different examples are placed on the board and the rule is given. The rule is now applied to the well-known problems in change of temperature during a winter day. The pupils solve these readily.

Now, and not until now, are the pupils ready to be told that another name for plus and minus numbers is positive and negative numbers, and that they have begun the study of the most important part of algebra, positive and negative numbers. The pupils are delighted to find that they understand so clearly what they had expected would be so hard.

This article is an account of what has actually been done with a class of about fifteen pupils in about twenty-five minutes. It is a description of a method that has been tried and proved successful. About four weeks after the lesson, the pupils were asked in an examination how they would write 1910 A. D. and 500 B. C. as positive and negative numbers. Everyone in the class answered the question correctly, although some retained the letters after they had prefixed the correct sign. The work has been presented in detail with the hope that it will help some teacher to answer the question, "How shall I make my work conform to the general principles I know so well, of making the subject interesting, practical, vivid, and clear?"

PUBLICATIONS OF THE CENTRAL ASSOCIATION.

A limited number of copies of the following publications of the Central Association of Science and Mathematics Teachers are still available. They may be had by writing to the secretary-treasurer, James F. Millis, 330 Webster Ave., Chicago.

APPENDIX OF PROCEEDINGS OF 1903 MEETING.

Contains report of committee on the correlation of mathematics and physics in secondary schools, with several valuable appendices. Price, 25 cents.

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Contains reports of addresses before general association and various sections. Includes report of committee on unification of secondary mathematics, report of geometry committee, and report of committee on algebra in secondary schools. Price, 50 cents.

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Includes address on sex segregation in high schools by J. E. Armstrong, report of committee on relation of elementary school nature study to secondary school science, reports of committees on fundamentals in biology, chemistry, and physics, report of committee on real applied problems in algebra and geometry, second report of committee on unification of secondary mathematics. Price, 50 cents.

ON THE SELECTION OF TOPICS FOR ELEMENTARY ALGEBRA.[1]

BY E. R. HEDRICK,
University of Missouri.

INTRODUCTION. The selection of topics for elementary algebra is one of the purposes for which this association was formed. Its solution cannot be glibly given. Among those who have devoted their energies to its study are some of the most brilliant teachers and scientists who have ornamented the history of American teaching. That I should arrive at any final decision beyond those established results upon which all the serious contributions to the subject in recent years agree, would be next to impossible. Rather I shall content myself with a restatement of some universally accepted principles propounded in recent years and I shall try to analyze the fundamental reasons upon which they are founded.

I refer in particular to the work of Klein in Germany, of Borel and Poincaré in France, of Perry and his followers in England, and to the following American reports: (1) Committee of the American Mathematical Society, the BULLETIN, November, 1903; (2) A Committee of the Central Association, SCHOOL SCIENCE AND MATHEMATICS, February, 1909; (3) A Committee of the Missouri Society, SCHOOL SCIENCE AND MATHEMATICS, April, 1908; (4) The Illinois Report, separately published, obtainable from the High School Inspector of the University of Illinois; (5) The Wisconsin Report, separately published, obtainable from the Inspector of High Schools of the University of Wisconsin; (6) H. L. Rietz, the National Education Report, 1909; (7) various articles in SCHOOL SCIENCE AND MATHEMATICS.

This by no means exhausts the recent contributions to this subject. I should refer in particular to the excellent text-books on the teaching of mathematics by Professor Smith of Columbia University and by Professor Young of Chicago University. The fresh consideration of these problems and the mere restatement of those principles which are most widely accepted is little less than a solemn duty if we are to profit by such studies, if we are to remain abreast of the times in which we live.

The purpose of the study of algebra. It is usual to say that the especial value of mathematics teaching is to exercise and to

[1]An address delivered before the Kansas Association of Mathematics Teachers, Oct. 21, 1910.

increase the power to think precisely, the ability to reason correctly, the habit of exact, concise expression, the appreciation of niceties.

As generalities, these values may well stand as expressing in a broad way some of the more extensive, indirect results of a mathematical training; but we have been led to believe by eminent psychologists of the modern school that the training of such broad powers cannot be the function of any one branch of learning, that the power to think precisely means only power in the realm of the topic concerned, at most, to borrow a political phrase, in the *"sphere of influence"* of that topic. To this difficulty in the transfer of a general faculty of the mind from one topic to another I shall recur; just now I shall point out only that it furnishes a sound basis for one established conclusion; that the course in algebra should include as wide a range of concepts, especially from among those with which the student has come or is to come in contact, as is consistent with proper intensiveness of study.

Valid as this conclusion is and sound as are the general indirect reasons for mathematical study stated above, it is upon the *direct* grounds for study that the teaching of any topic must finally stand. What are then the direct reasons for a study of algebra? Granted that one learns logic, neatness, and precision indirectly—even as a necessary concomitant—of a course in algebra, the real reason for teaching algebra or any part of algebra must always be some intrinsic virtue of each separate topic which distinguishes it from all the rest of human knowledge in its direct value and influence on the mind.

The solution of problems is such a direct purpose. Many problems which can be solved by algebra cannot be solved by any other portion of human knowledge; but if algebra is to rest upon its utility for these problems, the problems must themselves exist and need solution, it is not sufficient to invent them arbitrarily.

That this real ground exists, that it is at once one of the reasons for the invention of algebra and one of the present uses of algebra, has long been recognized. I state it here principally as a concrete instance of a direct value in distinction from those indirect values which I first mentioned. Such as this is, are the values which I now seek; a direct value peculiar to algebra itself, one of the reasons for its invention, one of its principal present uses for mankind at large.

Direct values of this kind are not only the justification for instruction but they come close to being the only reason for *selection* in our required curricula. Mistaken are those who dream that selection of topics is possible when each man claims that his own subject is worth while "for its own sake." Such a claim, though often correct, must lead to a deadlock. Equally mistaken, however, are those who see one direct value and happy in their prejudiced minds for having found one, give up the search and prove thereby that they were interested only in finding a justification for a pre-existent prejudice, rather than that their search for real values was sincere.

Problems constitute a real direct value for the teaching of algebra. I propose to you that there are others. Indeed, I shall propose to you that there is one other which totally eclipses this one in its importance.

To some the symbolism of algebra seems an overpowering direct value. It is at least true that the beautifully perfected symbolism of algebra is one of its direct values. It enables us to express in a brief form the facts of science and the facts of everyday life and to work with these forms by abbreviated processes. Such a value may by no means be overlooked. But there are those whose whole view of algebra is this symbolism and to whom a statement is not algebra unless it contains symbols. I wish to insist strongly on the view that the symbols of algebra are nothing but a chapter in shorthand. Every conceivable statement which can be expressed by means of algebraic symbols can also be expressed without the use of other symbols than English words. Is this not true? Can any significance other than abbreviations be attached to these convenient symbols?

The habit of functional thinking. To shut the eyes to the real value of symbols is no more foolish than to imagine that this chapter in shorthand is the real subject-matter of algebra. May I not say, for example, that in many a traditional course the algebra could not be seen for the symbols? I propose to try to reveal the *algebra,* not as this rather insignificant chapter of shorthand but as a body of knowledge to which the shorthand is only an incident. To you as teachers this is possible, for you have been through the forest; you have gained perspective; you may from your advanced position look back on algebra as on a forest from an eminence and see it as a living thing, not as a dead mass of separate symbols no one of which inspires the mind.

I may, then, speak of quantity without the use of shorthand;

I may discuss the relation between quantities; I may consider operations upon them. Such studies of quantity constitute algebra. But I may ask, what are the kinds of quantity which really require other than an arithmetical treatment? What are the quantities which actually occur in life, in science? To characterize these quantities no one adjective apparently suffices, but this characteristic I may note as intrinsic in the nature of those quantities which do not yield to elementary arithmetical processes: the quantities which occur in nature and in science have the distinctive property of variation and the allied property of dependence upon the variation of other varying quantities. The weight of an object changes as it nears the earth and depends upon its distance; the volume of a gas and its pressure are in constant change and depend upon the temperature; the speed of a chemical reaction, such as rusting of iron, varies, and depends upon moisture, temperature and other physical conditions; the total cost of a commodity depends upon its amount in a very simple way; the volume of a solid varies as the cube of a linear dimension. Thus, I might multiply examples to the limit of your endurance. Can you mention one quantity in nature, or in science not invented by man, which is fixed, unchangeable, whose qualities and properties suffer no alteration? I know of none.

The chief direct value of algebra, in fact the real subject-matter of algebra, aside from the rather insignificant chapter of shorthand which I have mentioned, consists in the study of variable quantities, the relations between such variable quantities and the acquisition of the ability to control and to interpret such relations.

This view of algebra is really quite old; it is the real basis for all possible applications of algebra in the sciences, since there the quantities experimented upon are necessarily changing and since the very essence of science is to discover the precise dependence of changes in one quantity upon the changes in another; for example, that the force of attraction between two bodies varies inversely as the square of the distance between them.

The emphasis upon this principle in algebra has naturally arisen, however, since the modern psychologist has demonstrated the comparative futility of basing the selection of topics upon indirect values or upon formal discipline. The chief exponent of these ideas in algebra has been Professor Klein, but they have already received universal acceptance by enlightened scholars throughout the world. In France, Borel; in England, Perry and

his followers; in America, every recent report has recognized or emphasized this view of the subject.

Stated in popular language, the standpoint taken is that those topics are certainly worth while which teach directly the appreciation of, and methods of control over the manifold relations which may exist between quantities, in particular between varying quantities. This, Klein has called *the habit of functional thinking.*

The insufficiency of the problem motive. Before this view was definitely formulated and generally accepted, a group of pioneers, recognizing the failure of formal discipline and of indirect motives as a basis for selection, had urged that the motive of problem solving should be made the chief basis for algebraic work and for selection of topics in algebra. Indeed books now in use, published within the past few years, state this notion positively and with no reserve whatever. So far as I know, no text-book has, however, actually carried to its logical conclusion this alarmingly destructive principle. For, carried out, this view would mean the entire elimination of those topics in algebra not concerned with the solution of problems.

To be sure, so-called practical problems may be devised *ad libitum* to serve as a pretense of application for any desired topic. Such problems abound; it is one of our most serious duties to rid the course in algebra from the ridicule which attaches to the type of problem recently made notorious in the example, "How old is Ann?" This duty has been very earnestly undertaken by a committee of the Central Association on the discovery of real problems; their work has been appearing in SCHOOL SCIENCE AND MATHEMATICS for something more than a year. It is not my purpose to comment here at length upon their very excellent work; rather I shall call your attention to the significance which such a search has for us here.

In the first place, this search signifies the presence in the traditional text-book of a variety of problems which are thoroughly bad. Consider, for example, the fact that a large percentage of traditional problems have the property that the data could not conceivably be known (in real life) until after the answers were known! Such problems, when presented in the guise of practical applications, are certainly hypocritical. They have assisted materially in disgusting many a child with the whole subject. While I should scarcely be able to demand that all problems be real, in the strict sense defined by the committee mentioned above, I do feel that we should all absolutely refuse these hypocritical problems.

I do not wish to dwell upon this mortifying condition of affairs. The conclusion I wish to draw from it and from the work of the committee mentioned is that real problems are desperately far from abundant in the majority of the topics in elementary algebra. In the case of simple equations in one or in two unknowns, a fairly adequate supply exists. In quadratic equations, a few exist. For simultaneous quadratics, the entire visible supply is hardly adequate to fill a page. For linear equations in three unknowns, radical equations, and for all the numerous remaining topics usually included in a course, few if any real problems within the student's grasp are to be found even after a most diligent search. As a direct motive for the extended study of algebra in secondary schools, the problem motive is thus woefully insufficient. A curious effect of the animated search for real problems has been the not unmerited conclusion on the part of educators uninterested in mathematics that algebra is of dubious value if this be the only direct motive and if real problems are so scarce that a protracted search is necessary to discover enough of them to last through a single course. However, no algebra has yet carried out the destructive program of omission of topics for which problems cannot be found. I have already indicated the belief that we must for the present retain many problems that are not strictly real, though surely we do not need the hypocritical ones.

The utter failure of symbolism as a direct motive. Under the reign of formal discipline, many a course was dominated by pure symbolism, without so much as search for any value, save that which was fondly supposed to result from the work itself because it was work. Now that this amazing doctrine has been laid to rest with other superstitions, except of course by those to whom the past is sacred, it appears that much of the work which was purely formal manipulation has no support. Thus all our recent reports condemn the Euclidean process for highest common divisor, though this process is, of course, of great importance to adults in the study of purely theoretical higher mathematics. Cube root has been abandoned for the same reasons. Square root remains by suffrance yet a little while. Complicated problems in complex fractions, in factoring, in radicals, in long division, are a thing of the past, still treasured only by those whose faces are toward the past. Simultaneous quadratics no longer form one of the central chapters, with a maze of special cases to be solved each by its special clever device of Chinese puzzle character.

Logarithms are now wholly for the purpose of computations; the base is always ten; those disciplinary theorems regarding the effects produced by change of base are gone. Infinite series has left little or no trace of its former awfulness—shame that it was that this schoolboy's horror was particularly difficult because the text-book was actually wrong, as are to-day the so-called "proofs" of the incommensurable cases in geometry.

These and other actually achieved changes from the traditions of fifteen years ago prove that the mathematical world has been by no means asleep to the advance in modern pedagogy. But these changes are certainly not all accomplished. Elimination of other topics will go on; the exercises in the topics which remain will become less complex, probably less numerous. In particular the complex examples in factoring ought to be weeded out quite severely, leaving, of course, the *simple* cases. Long division should be greatly minimized. Square root of algebraic expressions might well follow cube root to oblivion, or at least to an appendix! The treatment of radicals must certainly be altered rather violently.

Only in the last paragraph have I allowed myself the liberty of prediction, and of expressing personal conviction. Otherwise, the changes I have mentioned are to be classed as present accomplishments; they are all accepted and in operation by the majority.

Symbolism as a direct motive, formalism, has proved an overwhelming failure. With it has disappeared (let us hope forever) an enormous mass of mechanical manipulation of meaningless hieroglyphics. There remains only the sane use of symbols as abbreviations for actual meanings which we would express. Shorthand has come into its own; the wreckage of all that the old disciplinarian saw in algebra is but a single chapter of shorthand. Meanwhile, algebra has emerged, as from a chrysalis, from the entangling maze of dead marks, more beautiful and wonderful than ever even dreamed those who saw only the dry shell, believing that was all; who never knew the living thing within, the life and very reason for existence of the shell, which, freed from its maze, shows itself incomparably superior.

Algebra and functional thinking. The life of algebra is characterized by the quality which pervades all life, variability. The spirit of algebra is the dependence of varying quantities upon each other. The dependence of one quantity upon another is a phenomenon which has escaped only the most untutored

savage. The common phrase "cause and effect" implies precisely such a dependence in all of the myriad instances in which that phrase is used. The tools for the precise control of cause and effect are none other than algebraic relations. Whether these relations be stated in symbols or in ordinary English is quite beyond the point; surely no serious person can attribute vital significance to a mere shorthand.

While cause and effect remain vague and unformulated, there can be little control over their relation. This is the actual situation of the whole race toward those scientific problems which have not yet been solved. It is the actual situation of all beings to whom algebraic relations, that is, precise statements of the relations between variable quantities, are unknown. A familiarity with algebra means a familiarity with means of such precise expression, an appreciation of the various possibilities which actually occur in such relations, the power to control and interpret them.

That the untrained individual, that is, the person ignorant of algebra, cannot think of such relations readily is strikingly shown by Sir Oliver Lodge in that excellent book, "Easy Mathematics, Principally Arithmetic." He points out that most persons will attempt to use proportion in cases to which it is not adapted; and cites his own experience in testing persons with such ridiculous problems as these: "If a camel can go without water for ten days after drinking fifteen gallons, how long could he go if he drank one hundred gallons?" "If a boy can slide eighteen feet on the ice with a running start of twenty feet, how far could he slide after running half a mile?"

The same point enters many a story, where the moral is that if a little of a medicine is good, more is better. How many people know, for example, that a cause which produces an effect may not, if increased, show a further increase in that effect? A study of such relations as $y = x^2-2x+5$ would tend to establish the possibilities of variation firmly in the consciousness.

This motive runs through algebra in a surprising manner. Whenever one quantity y depends upon another quantity x in such a way that when x is known y can be determined, y is said to be a function of x. This is only an accurate statement of the ordinary "cause and effect" situation. In algebra, practically all the forms used are of this kind. Each expression is such that its value is calculable when the value of the quantity (or the values of the quantities) which it contains, is given.

The study of the possibilities of such expressions runs throughout algebra. Scientific formulæ which employ algebra are all distinctly of this class; every such formula states one quantity in terms of another or others, the very reason for its existence being the calculation of the value of the first quantity when the others are known. The way in which the first quantity changes when changes are made in the others is precisely the thing the scientist wishes to know and to control.

In algebra proper, beside all the pervading use of such function forms, we may especially mention the chapter usually devoted to proportion, which is nothing but the simplest possible kind of relation between variables; the chapter on variation itself, which has for decades held its own as an integral part of algebra, and needs only amplification to meet fully the awakened demand for the fuller appreciation of its meanings; the topics involving substitution, which actually means the calculation of one quantity in terms of others; the topics involving tabulation, such as logarithms, which actually state in full the values of one quantity in terms of another; the topics involving graphical representation, which have recently come into full and universal recognition.

All the topics here mentioned, particularly variation and graphical representation, are absolutely identified with this view of algebra. All of them assist materially in forming an appreciation of, and gaining the ability to control and interpret, the relations between interdependent varying quantities. All of these topics deserve great amplification, though care should be taken to avoid rigorously any complication, or the use of complex groups of symbols.

Graphical representation seems to have won its way to absolute acceptance. It, more than any other topic, shows clearly the ways in which one quantity may vary owing to its dependence upon another. It is used universally and with the greatest profusion by students of all of the sciences, to express these very relations. It was actually first used in secondary schools—be it said to our everlasting shame—by classes in physics! I think there are probably still schools in which graphical representation on cross section paper is used in classes in physics, but not in the classes in algebra. This is, of course, a condition of affairs which cannot conceivably continue.

With this motive, then, firmly established, algebra receives for the first time a thoroughly firm foundation in modern peda-

gogy; for we have seen that neither the shorthand of symbolism, nor the search for problems affords a satisfactory basis. Moreover, algebra itself emerges strengthened and beautified, no longer needing any apologist, but manifesting itself as a true need of the modern world, which is, both in its manifold scientific enterprises and in its everyday affairs, vitally interested in controlling and interpreting the relations between varying quantities, and in assuring itself that even its humblest citizens have some appreciation of the possibilities of such relations. As a means for selection of topics, this modern view of algebra is therefore absolutely satisfactory.

MATHEMATICS AND IDOLATRY.

By G. A. Miller,
University of Illinois, Urbana.

The ancient Babylonians represented each of their gods by an integer between one and sixty, which exhibited his supposed rank in the heavenly hierarchy. They possessed also a prophesying geometry in which they employed, among other figures, a pair of parallel lines, a square, and a certain figure with a reëntrant angle. The Egyptians possessed, at least as early as 1700 B. C., a table of unit fractions giving the value of 2 divided by every odd number greater than 3 and less than 100 in terms of fractions having unity for a numerator. As instances of this table we may cite:

$$\frac{2}{7}=\frac{1}{4}+\frac{1}{28},\ \frac{2}{13}=\frac{1}{8}+\frac{1}{52}+\frac{1}{104},$$
$$\frac{2}{25}=\frac{1}{15}+\frac{1}{75},\ \frac{2}{89}=\frac{1}{60}+\frac{1}{356}+\frac{1}{534}+\frac{1}{890}.$$

Although such decompositions have very little practical value they occupied a significant place in the development of mathematics during several thousand years, having been used by the Greeks, Hindus, Arabians, and receiving considerable attention even in such an important work as the *Liber Abaci* of Leonardo of Pisa, published in the early part of the thirteenth century.

Other evidences of undue reverence for ancient practices in mathematics are very numerous. The general acceptance of 3 as the ratio of the circumference of a circle to its diameter in central Asiatic literature, including parts of the Bible, and the persistent use by early surveyors of the incorrect ancient Egyptian

formula for the area of an isosceles triangle, as the product of half of the base and one of the equal sides, are among the best known instances of the long-lived and widespread uses of clearly incorrect mathematical formulas. On the other hand, the strange and unreasonable attributes assigned to various integers by the Pythagoreans and the later Greeks, and the belief, even as late as the sixteenth century, in the efficacy of plates inscribed with magic squares and worn on the body are evidences of an undue reverence of numbers and numerical relations. Such undue reverence is also exhibited by the gematria of the ancient Jews and others, who used the letters of the alphabet to represent numbers and associated with various words the numbers represented by their letters. The Jewish year of 355 days, for instance, was due to the fact that the letters of the ancient word for year represented the number 355.

The extensive excavations at Nippur, under the direction of Professor Hilprecht, have exhibited how prominently the number $60^4=12,960,000$ enters into the multiplication tables of the ancient Babylonians, and have thrown additional light on the fact that this number plays also an important rôle in the philosophy of Plato, "measuring the cycle of uniformity." Even at the present time there are many who seem to fear that special numbers, such as 13 and 23, have some peculiar mystic attributes and this fear may be a relic of the wonderful grasp number mysticism has had on the minds of uncivilized nations.

As many of the elements in the intellectual development of the world are reflected in miniature in the intellectual development of the individual, it is natural to inquire whether the evil effects of excessive reverence for inherited methods and results, and the undue attention to matters of secondary importance which retarded the development of mathematics in general are also evident in the mathematical development of the individual. It appears to the writer that they are and that we have here a tendency which constitutes one of the most serious problems of the teachers, extending from the grades through the university.

The child begins by idolizing the knowledge of the teacher and the author of the text-book. One of the most important attitudes for real mathematical progress is to exclude from one's own store of knowledge everything except such results as one has traced back to postulates or definitions. The fact that the teacher said so or that the text-book says so merely calls for a careful consideration of the matter and not for its

immediate acceptance. This attitude should become more pronounced as the student advances and its cultivation is one of the chief functions of the true teacher.

The cultivation of this attitude calls for more devotion and skill than one might at first suppose. Teachers share with most other people the desire to be regarded as wise and the pleasure which implicit confidence conveys. They naturally feel the need of respect and confidence on the part of their pupils in order to make their work as effective as possible, and yet few things can make mathematical instruction less effective than a confidence devoid of a searching scrutiny as to the reasons of every statement, and a careful determination of the limit within which it may be regarded as true, if it is one of those broad statements of partial truths which are generally regarded as essential to life and real interest in certain mathematical doctrines. From the nature of his position the teacher is called upon to express an opinion on a great variety of questions and it is desirable that he should do so. These opinions, even when they need modifications, are probably the closest approximations to the truth that could be comprehended by the average student and they often awaken helpful thoughts on the part of those who are inclined to probe things deeply.

As a student advances he will naturally have more reverence for mathematics as a subject and comparatively less for individual men, books, or schools. While great honor is due to particular individuals, yet when compared to the whole field their work must appear small. Mathematics is too big for one man or for a half a dozen of men, and if an advanced student associates mathematics with a few names or even with a few schools he assumes a benighted and idolatrous attitude which invites the rebuke of all true devotees of the subject. A great teacher is a great source of inspiration and leads his students into great subjects—so great that they forget most of the words of their teacher but remember gratefully the guiding influence. He is really a grateful student who develops the lines of thought dear to his teacher far beyond those places to which he was carefully guided, and he who merely reflects what he has received from his teacher does not deserve the name of true student.

The desire to receive instruction from renowned teachers deserves encouragement but it should not be forgotten that many of the leading mathematicians did not enjoy such advantages. If anyone tries to receive unusual credit for having studied

under a renowned teacher it would be well to ask him who was the teacher of his teacher. It would frequently turn out that his teacher had gained renown without enjoying any unusual advantages. This is probably more commonly true in mathematics than is most other subjects in view of the fact that the truthfulness of claims of useful advances in mathematics is generally open to definite proof. It has therefore been possible for Grassmann, who "studied under no one,"[1] to rise to such eminence, even if his work was not recognized as promptly as it probably would have been if he had been more closely associated with eminent mathematicians during the early part of his career.

Whenever the teacher, text-book, or school receive undue attention on the part of the student he is in the grasp of a kind of idolatry. These are very important, but the thing of much greater importance is an interest in the general subject of mathematics. Teachers, text-books, and schools should tend to awaken this interest and should point away from themselves to this big and inexhaustible mine. In many cases this interest has been awakened by literature which accidentally fell into the hands of the student. For instance, Lagrange, one of the greatest mathematicians of all times, became interested in the subject through a memoir of Halley which he found by chance. Fortunately our mathematical libraries are becoming better at a rapid rate so that mathematical students are more commonly exposed to a deep interest in the subject instead of a superficial interest in the guide posts pointing to this subject.

[1] Bulletin of the American Mathematical Society, Vol. 16 (1909), p. 115.

HELPS FOR TEACHERS OF SCIENCE.

There has just been issued (November 8, 1910) a revision of Circular 94, Office of Experiment Stations, United States Department of Agriculture. This circular contains a most excellently catalogued list of the various free publications that are most helpful to teachers. The bulletins are classified with reference to education, plant production, animal production, agricultural technology, engineering and economics, and with reference to teaching the different sciences. Every teacher in any science in secondary schools should have this circular and should use it in securing some of these valuable means of freshening and making more vital the work of the text and laboratory.

O. W. C.

MATHEMATICS A GOOD TONIC.

There is no better tonic for young minds than drill in mathematics. We should work against the tendency to make school work "soft."—Ex-President Duncan McGregor, Platteville State Normal School.

TWO YEARS' PROGRESS IN MATHEMATICS IN THE UNIVERSITY HIGH SCHOOL.

By G. W. Myers,
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This report relates to the school years 1908-09 and 1909-10. Its late appearance is due to the writer's absence in Europe from the School of Education during the school year 1909-10. It is thought that some will be interested to hear even at this late date of the progress in pursuance of the experiment referred to in the issues of the *School Review* of October, 1908, and February, 1909.

Teachers of mathematics who are students of the history of education readily assent to the view that while antiquity educated the individual for the *state* and the middle ages educated him for the *church,* modern times aims to educate the individual for *himself.* The difficulty we find arises in practically administering this theory of modern education through the mathematical subjects. Through many years of critical scrutinizing of the mathematical subjects as self-contained domains of truth by many of the world's keenest thinkers, the various mathematical branches have acquired for themselves a high degree of self-sufficiency and independence. It was not thus that the subjects grew up under the race struggle for knowledge and power. So little attempt has been made to interpret mathematical history as an aspect of race development, to bring out its human significance, to make plain the meaning of the school subjects of mathematics in the social whole of human life, that it is still far from easy to recognize practically the interests of the individual as the control of the acts and the matter of mathematical teaching. The separate subjects now lie before us in highly isolated form and to reëstablish mutual connections, connections laterally with kindred subjects and with the interests and concerns of youth without impairing the integrity and scientific merit of mathematics as mathematics, is a problem of capital educational import and of extreme difficulty and delicacy. It is, however, preëminently a problem for teachers. The problem of finding something better calculated to contribute to the needs and interests of the individual, something more nutritious than overmature formal logic, was the point of departure of the University High School experiment begun several years ago.

During the year 1908-09 the teachers have entirely recast

the book of preliminary notes on *First-Year Mathematics* in the light of three further years of class room experience in the University High School and elsewhere into the text entitled *First-Year Mathematics*. This book was issued by the University of Chicago Press in September, 1909. It is intended for students of the first year of secondary schools. As completed it presents in teachable form an interweaving of the more concrete and easier phases of the first courses in both algebra and geometry. The chief emphasis is placed on algebra, but a considerable body of related fundamental notions and principles of geometry is woven in, while some advances are made in one direction or another upon the most concrete, graphic, and practical aspects of elementary geometry. Geometrical treatments are at first intuitive, inductive, and experimental, but in some instances these are followed by the quasi-experimental methods of superposition. A like informality as to method characterizes the earlier part of the algebra. The transition from the informal procedure of the earlier part to the formal procedure of the later part is gradual. The reasoning becomes more and more highly deductive throughout the latter half of the book. This accords with both class room practice and *a priori* reasoning as to the normal procedure for secondary mathematical instruction. During the year 1909-10 the text has been found to work to the entire satisfaction of all the teachers, and results have been very gratifying indeed.

Some may not know that the final form of the text just alluded to has been seasoned through four years of concerted and critical class room use. Six high school teachers with experience ranging from six to twenty-six years have participated in the authorship of the text and in the most searching class room tests. The sole purpose all along was to get something that would teach mathematics without discouraging the pupil or destroying his interest and faith in the subject. The marked improvement wrought in the mathematical tone and attitude of the early classes of the University High School mainly, we think, through the influence of this book, justifies strong claims for the general suitability to such students. Finality is not claimed even for its present form. No form can be final with progressive teachers.

As soon as the manuscript of the revised first-year book was in the printer's hands the teachers proceeded to the preparation of the *Second-Year Mathematics* to follow the former text. Two or three conferences a week were given to hearing and discuss-

ing plans for the second-year text in the light of the experience derived from the use of the first-year book. The following outline of agreements was approved by the conference and the authors set to work at once on the manuscript under their guidance.

General Agreement for the Guidance of the Authors in Treating the Materials of Second-Year Mathematics.

I.

The aim of *Second-Year Mathematics* is threefold:

1. To complete with sufficient rigor and fullness for secondary school purposes the rest of the essentials of plane geometry and of a first year course in secondary algebra that is either not treated at all or is treated with insufficient fullness in *First-Year Mathematics.*

2. To develop an interest in the study of the science of geometry and some power to apply geometry through exercises in construction, through practical problems and applications in geometric, algebraic, and non-mathematical fields.

3. To secure logical connection and unity to the subject-matter and to develop the space intuitions and the power of mathematical reasoning through a carefully chosen and a considerably reduced number of principal or major propositions. These major propositions are to serve as genetic centers for the geometry of the year's work. In this sense the geometry is to be the unifying bond of the second-year text.

II.

To avoid misunderstanding and circumlocution the following terms are used in this outline (not in the book) in the senses here described:

1. *Practical use* means some sort of measurement, or experimental or practical, or construction problem that may arise in out-of-school and in-school life, that exemplifies the use of some geometric truth, but that may be solved to the satisfaction of the pupil and with sufficient rigor for practical purposes *without an antecedent knowledge of the geometric truth.* In our text a *practical use* is to serve as an *approach* to the theoretic establishment of the truth which it exemplifies.

2. *Practical application* means a more difficult or a less intuitive, problem of the same general character as the *practical use,* but to be solved after the theory has been developed, and to serve as an *application* of the theory. A practical application is to drive home the purport of a geometric truth to mark its

meaning and scope more sharply, and to enliven the interest in it through an immediate appeal to its practical or scientific worth.

3. *Exercise* means what we usually understand (a) by proof of a construction; (b) by a corollary; (c) by an original, geometric or algebraic-geometric; or (d) by a numerical exercise.

4. The term *corollary* shall be sparingly used.

It is agreed that in the first three or four chapters of the book treatments of the principal propositions shall plainly feature, though not slavishly observe the following order of steps:

1. Practical use, aiming halfway to lead the learner through measurements or constructions or the exercise of the intuitive judgment to a close guess at the substance of what is immediately to be formulated and demonstrated.
2. Demonstrations. Here the pupil is to be made to rely on himself for as much of the reasoning as the author deems practicable, the understanding being that the conference favors a plan of detailing written demonstrations that tells as little and elicits as much as is practicable.
3. Practical applications and exercises (say three to eight).
4. Algebraic exercises (three to six).

As the work proceeds, step 1 (the approach) may well become of a more and more purely geometrical character, or it may be elided entirely where it is believed safe or advisable to rely on the geometrical interest purely. When some particularly happy application of a geometrical truth to a problem of practical life can be found, it is the opinion of the conference that such applications should be given even to the end of the year's work.

III.

The following scheme exhibits the assignment of the algebraic subjects to the chapters to which it is believed they are most naturally associated. The author of the chapter in question will understand that these algebraic subjects are assigned to him for (1) informal and intuitive treatment, or (2) for topical treatment according as he decides the one or the other is the more advisable, subject of course to subsequent determinations of the conference in passing the author's work under revision for publication.

The author of any chapter is free to use the algebra of *First-Year Mathematics* and of any topic associated in the following outline with a chapter that precedes his own chapter. If an author is in doubt about the extent to which he may draw upon

a knowledge of an algebraic subject associated with a chapter preceding his own, he must consult carefully the author who is charged with primary responsibility for dealing with this subject.

OUTLINE OF CHAPTERS AND ASSOCIATED ALGEBRAIC WORK.

	CHAPTERS.		ASSOCIATED ALGEBRA.
I.	Congruency of Rectilinear Figures and Circles.	I.	Use of Algebraic Notation and of Equations.
II.	Ratio, Proportion, Similar Triangles.	II.	Graphing Circles.
III.	Measurement of Angles by Arcs of Circles.	III.	Use and Reduction of Radicals.
IV.	Similarity and Proportionality in Circles.	IV.	Quadratic Equations; Solution by Formula.
V.	Inequalities in Triangles and Circles.	V.	Inequalities; Indeterminate Equations; Discussion of Roots; Simultaneous Quadratics.
VI.	Areas of Polygons.	VI.	Use of Algebraic Formulas and Equations.
VII.	Regular Polygons in and About a Circle.	VII.	Use of Formulas and Equations.
VIII.	Geometric Algebra.	VIII.	Radical Equations.

With the exception of the caption beside VIII above, the chapter headings of *Second-Year Mathematics* are the captions of the first column. The last chapter was finally changed to *"Problems and Exercises in Graphic and Geometric Algebra"* as being more closely descriptive of the work done in this chapter.

The thought of the conference was that while the everyday uses and practical values are the surest and the most direct routes to an early interest in the science of geometry, the main mathematical goal of these routes is a real and a self-sustaining interest in the scientific phases of the mathematical subjects. The conference accordingly believed that the educational ends of second-year mathematics would be best subserved by placing a gradually abating emphasis upon purely practical and experimental matters and a gradually increasing stress upon geometric procedure. It was not believed, however, that practical uses should be altogether excluded even in the final portions of the book, *Second-Year Mathematics*.

The late appearance of this article makes possible a statement of the nature of *Second-Year Mathematics* which was issued from the University of Chicago Press in June, 1910. Preprints of the book were used during the latter part of the school year 1909-10 and are being used this autumn, so that something may also be said about the teachableness of the final form of the manuscript.

The book carries forward through the second high school year the mixed type of material and the plan of treatment of the *Revised First-Year Mathematics.* The two texts together cover well the essentials of what is commonly required of all pupils of the public high schools and it includes in addition to this the elementary notions of plane trigonometry through the solution of right triangles and a fuller treatment of quadratic equations than is commonly given before the third high school year. Each book constitutes a well-balanced and not over-heavy year of work.

The second book lays chief emphasis on geometry as did the first book on algebra. To take up the work of these texts then requires no great departure from the order of subjects now prevailing in secondary curricula. The plan of the books enables the work of the first year to connect smoothly and strongly with eighth grade work through both mensuration and general number, rather than through either topic alone, as is customary.

Toward the close of the first-year book geometrical ideas are refreshed and brought into the focus of attention, and some preliminary geometrical work is done. This gives the pupil an inkling of what is to come from the vantage point of what is most useful and alluring to him and it is thought that the treatment is such as to impress him that the next year would be pleasurable and profitable to him mathematically.

The second book then begins with some constructive and inductive geometry, passes soon to easy demonstrative geometry, employing for a time the half experimental method of superposition. At no place are the algebraic notions already acquired allowed to remain long unused. By the use of algebraic notation and the continual application of the equation to geometrical matters, the hold on algebra is kept firm until the opportunity arises to complete the solution of quadratics and to discuss the roots with profit. The quadratic equation is used from time throughout the second year and plane geometry is taught with nearly or quite its usual fullness.

Though published primarily for use in the University High School, since this school possesses no features calling for material or method in any way different from that needed by all good public high schools, this text is believed to be of interest and service to all high school teachers who are concerned with improvement of their work. The results with us so far as can at present be judged are most excellent and distinctly superior to what could be gotten through standard texts

During the year 1908-09 the second-year classes used a standard text in geometry, carrying along with the work of the text associated algebraic problems and exercises from the little book, *Geometric Exercises for Algebraic Solution.* The exercises of this book are collected under such captions as complementary angles, supplementary angles, right triangles, isosceles triangles, scalene triangles, parallel lines, etc. This makes it easy for the teacher while using some standard text in geometry to select algebraic exercises that relate to the bit of geometry under immediate study. The teachers gave from one to two and a half hours a week to this book, some distributing the time and others blocking it into a single day. Each teacher followed in details the plan that seemed best to him, it being believed that out of variety of experience comes wisdom.

The teachers without exception reported favorably, some in the warmest terms on the procedure of carrying, even in this somewhat mechanical fashion, some algebraic work along with the work in plane geometry.

This plan is commended to teachers of plane geometry as a practicable means of using the adopted text and at the same time trying some of the modern ideas of simultaneous algebra and geometry. This plan has the virtue, without disturbing adopted programs, of supplying means of holding, through the usual course of plane geometry, the ground made in algebra during the preceding year. It furnishes a little relaxation from the too great concentration on one narrow field of mathematical ideas—highly abstract ideas, moreover—at so early an age that the pupil cannot profit by it.

Recapitulation.

To recapitulate we recall that three years ago we tried in the second-year classes a plan of paralleling the plane geometry course with a course in algebra, giving three periods a week to the geometry and two to algebra, with results unsatisfactory to both teachers and pupils. A more whole-hearted and genuine trial would, however, have led to better results. Two years ago we tried a plan of parallel geometry and algebra, distributing the time roughly in the ratio of three to two, but undertaking to select at suitable times such parts of algebra as coördinated somewhat plainly and strongly with the geometry in question. Results were more satisfactory to pupils but were still unsatisfactory to the teachers, mainly on account of the lack of suitable

text-book material. The next step was to compile and arrange the supplementary exercise book, *Geometric Exercises for Algebraic Solution.* Results were fairly good, but not good enough.

We then passed to the mixed geometry-algebra material with primary stress on the geometry of *Second-Year Mathematics.* It now seems that the steps by which we arrived at the present *status* are both natural and logical. The steps have furnished all participants with a rich fund of professional experience.

We feel also that our claim that the present status and plan are the outcome of practical class room experience, rather than of *a priori* reasoning, will not be denied. In so far as experience indicates anything it points to a decided preference in practical teaching for the mixed type of mathematics. We detect in our pupils the following effects:

The increase in mathematical interest, earnestness, and spirit is marked.

A genuine belief, attained earlier than formerly, among pupils as to the real worth of mathematical study.

An improvement in independence and solidity of mathematical thinking among pupils of the first and second years.

Pupils seem to feel less than formerly that they are to learn a mathematical subject merely to pass an examination and then to forget it all.

Pupils try more frequently to check and guarantee algebraic reasoning and results by some sort of geometrical picture.

A considerable reduction in the percentage of failures of first and second-year pupils under the new plans.

Pupils ask in *friendly* interest more frequently than formerly what advanced subjects are like with the manifest desire to take more rather than less mathematics.

Most teachers will agree that these results are wholesome and indicate a distinctly tonic influence.

WORK OF THE THIRD AND FOURTH YEARS.

As announced in former reports referred to above, the third-year work in mathematics consists of two parallel courses, one in plane trigonometry and the other in advanced algebra, or algebra through and beyond quadratics. During the first half-year trigonometry receives three hours a week and advanced algebra two hours. During the second half-year the ratio of time distribution is reversed. This plan now three years old, of continuing the plane geometry into plane trigonometry, rather than into solid geometry, is working with gratifying results.

The work of the third year is required of pupils seeking certification to technological schools having strong entrance requirements in mathematics. With all other pupils the third and fourth-year work is optional, excepting a special review course in the fourth year of one hour a week for students seeking the University High School diploma.

Students who seek advice of teachers as to the best general high school training in mathematics are advised to take the third-year course, especially the trigonometry. The large number of pupils taking these courses is evidence at least that they are popular. The high character of the work and the comparatively small percentage of failures in these classes is evidence of their interest and value to the pupils.

The parallel, but separate, courses of the third year permit the pupil who has had the mixed type of mathematics of the first and second years, to see the branched type of mathematics as geometry, algebra, and trigonometry. It should be said, however, that the longer these parallel courses are continued the more do teachers discover profitable ways of unifying the algebra, higher geometry, and trigonometry. The courses tend naturally to grow together. Accordingly the question soon to be taken up by the mathematical faculty is whether the plan of parallel courses shall be continued, or shall be replaced by unified work of the mixed type. It is expected that a later date may bring out some experimental findings on this question.

In conclusion, the writer speaks freely about the work of the high school classes because the practical results have been worked out by other teachers. While he has participated in all the conferences and has assisted in reducing the wishes of the conferences to writing, the mathematical classes of the College of Education have consumed all his teaching time. He feels that whatever disadvantages this may have had to the writer himself, it had the compensating advantage of enabling him to hold a somewhat more completely objective point of view toward the work than would otherwise have been possible. He feels therefore that this rather external relation to the practical side of the experiment should partially protect him from the charge of having so high a subjective interest in the practical issues of the work as to deprive his judgments of that objective quality which alone can give them value to the school public.

PROBLEM DEPARTMENT.

By E. L. Brown,
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Readers of this magazine are invited to send solutions of the problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to E. L. Brown, 3435 Alcott St., Denver, Colo.

Algebra.

217. *Proposed by W. T. Brewer, Quincy, Ill.*

Solve for x, y, and z:

$$8x+8\sqrt{xy}+7y=600 \qquad (1)$$

$$6z+8\sqrt{yz}+7y=520 \qquad (2)$$

$$z+3\sqrt{xz}+3x=210 \qquad (3)$$

Solution by H. E. Trefethen, Kent's Hill, Me.

From (1) $\sqrt{x}=-\frac{\sqrt{y}}{2}\pm\frac{\sqrt{10}}{4}(120-y)^{\frac{1}{2}}$,

and $x=75-\frac{3y}{8}\mp\frac{\sqrt{10}}{4}(120y-y^2)^{\frac{1}{2}}$.

From (2) $\sqrt{z}=-\frac{2\sqrt{y}}{3}\pm\frac{\sqrt{26}}{6}(120-y)^{\frac{1}{2}}$,

and $z=\frac{260}{3}-\frac{5y}{18}\mp\frac{2\sqrt{26}}{9}(120y-y^2)^{\frac{1}{2}}$.

Substituting in (3) and reducing we have

$\pm 2(45\sqrt{10}+17\sqrt{26})\sqrt{120y-y^2}=18\sqrt{65}(120-y)+7320-29y$. After squaring both members, this equation reduces to the form $y^2+py=q$, from which the values of y may be found and substituted in (1) and (2) for the values of x and z.

218. *Proposed by J. A. Hardin, Ft. Bliss, Tex.*

If $x=7\pm4\sqrt{2}$, and $y=\sqrt{25+\frac{17}{2}\sqrt{2}}\pm\sqrt{25-\frac{17}{2}\sqrt{2}}$,

show that $2y^3=x^2\sqrt{17^2-x^2}$.

Solution by Orville Price, Denver, Colo.

$x^2=81\pm56\sqrt{2}$, $y^2=50\pm31\sqrt{2}$. $\therefore \frac{x^2}{y^2}=1\pm\frac{1}{2}\sqrt{2}$.

From $2y^3=x^2\sqrt{17^2-x^2}$ we have $\frac{x^4}{y^4}=\frac{4y^2}{17^2-x^2}$.

$$\frac{x^4}{y^4}=(1\pm\tfrac{1}{2}\sqrt{2})^2=\frac{3}{2}\pm\sqrt{2}$$

$$\frac{4y^2}{17^2-x^2}=\frac{200\pm124\sqrt{2}}{208\mp56\sqrt{2}}=\frac{3}{2}\pm\sqrt{2}.$$

Hence $\frac{x^4}{y^4}=\frac{4y^2}{17^2-x^2}$, and therefore $\frac{x^2}{y^2}=\frac{2y}{\sqrt{17^2-x^2}}$ or $2y^3=x^2\sqrt{17^2-x^2}$.

221. *Proposed by J. L. Riley, Waycross, Ga.*

Show that the product of any four consecutive integers plus one is a perfect square.

Solution by I. L. Winckler, Cleveland, O., and C. C. Spooner, Marquette, Mich.

Let x, $x+1$, $x+2$, $x+3$ be four consecutive integers. Their product plus one is

$$\begin{aligned} &x(x+1)(x+2)(x+3)+1 \\ &=(x^2+3x)(x^2+3x+2)+1 \\ &=(x^2+3x)^2+2(x^2+3x)+1 \\ &=(x^2+3x+1)^2 \end{aligned}$$

x^2+3x+1, the square root of the product, can be expressed in terms of the four consecutive integers in various ways: for example,

$x^2+3x+1 = (x+1)^2+x = x(x+2)+(x+1) = (x+1)\ (x+3)-(x+2) = (x+2)^2-(x+3)$.

$\sqrt{6\cdot7\cdot8\cdot9+1} = 7^2+6$, or $6\cdot8+7$, or $7\cdot9-8$, or 8^2-9.

Geometry.

219. *Proposed by M. H. Pearson, Montgomery, Ala.*

Construct a triangle, given the altitude, the median, and the angle bisector, all from the same vertex.

I. *Solution by R. E. Krug, Milwaukee, Wis., and L. E. Ling, La Grange, Ill.*

Construct a triangle, given the altitude, the median, the angle bisector, all from the same vertex.

Let the lines h, w, m, represent the altitude, angle bisector, and median, respectively.

At H, in the straight line XY, erect the perpendicular HZ to XY. On HZ, take HC, equal to h. With C as a center, and a radius equal to m, describe the arc cutting XY in M. With the same center, and a radius equal to w, describe the arc cutting XY in D. At M, erect a perpendicular to XY. Produce CD over D to meet the perpendicular erected at M in N. At C, erect a perpendicular to CN to meet the perpendicular at M in Q. On NQ as a diameter, describe the circumference, cutting XY in A and B. Draw AC and AB; then ABC is the required triangle.

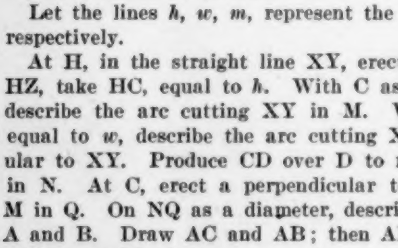
The proof is obvious.

II. *Solution by A. W. Rich, Worcester, Mass., and L. B. Mullin, Brooklyn, N. Y.*

Construct right triangle AHM with hypotenuse AM = median and AH = altitude; on HM find D so that AD = bisector; erect ME perpendicular to HM; prolong AD to meet ME at F; erect perpendicular bisector of AF to meet ME at O. With O as a center and OA as radius describe circle cutting HM at B and C. ABC is the triangle required.

III. *Solution by T. M. Blakslee, Ames, Iowa.*

Let AD, AL, AM be the altitude, angle-bisector, and median respectively. At any point D in the line RS erect the perpendicular DA. With A as a center and AL, AM as radii describe arcs intersecting RS in L and M. Draw AL, AM. Take P any point in AL and on AP as a diameter construct a circle. Through P draw A′P perpendicular to RS and prolong to meet AM in K. Through K draw chord B′C′ perpendicular to diameter AP. Also through K draw xy parallel to RS. Let AC′, AB′ prolonged intersect RS in B, C, respectively. ABC is the required triangle.



ABC has for its altitude AD. Since B′C′ is perpendicular to AP, ∠ PAC′ = ∠ PAB′. ∴ AL bisects

$\angle$ A. Also PB$'$ = PC$'$. $\therefore$ $\angle$ PB$'$C$'$ = $\angle$ PC$'$B$'$. (x lies on AC and y on AB.)

But $\angle$ PB$'$C$'$ = $\angle$ Pyx, since points P, B$'$, y, K are concyclic.

Similarly, $\angle$ PC$'$B$'$ = $\angle$ Pxy. $\therefore$ $\angle$ Pyx = $\angle$ Pxy, and $\therefore$ Px = Py.

But PK is perpendicular to xy, the latter being parallel to RS.

$\therefore$ K is the mid-point of xy, and hence M is the mid-point of BC. Therefore AM is the median from vertex A.

220. *Proposed by G. B. M. Zerr, Philadelphia, Pa.*

If c = chord and h = height of a segment of a circle, prove the following approximation, given in works on mensuration:

$$\tfrac{2}{3}ch+h^3/2c = \text{area of segment.}$$

Solution by H. E. Trefethen, Kent's Hill, Me.

O is the center of circle and r the radius. Chord AB = c. Arc AB = $2x$ (in radians). Then area of triangle AOB = $c(r-h)/2$, area of sector

$$\text{AOB}=r^2x=r^2(\text{arc sin } c/2r)= \frac{cr}{2}+\frac{1}{2}\cdot\frac{c^3}{3.2^3.r}+\frac{1.3}{2.4}\cdot\frac{c^5}{5.2^5.r^3}+\text{ etc.}$$

$$\text{Hence area of segment AB}= \frac{ch}{2}+\frac{c^3}{2^4.3r}+\frac{3c^5}{2^8.5r^3}+\frac{5c^7}{2^{11}.7r^5}+\text{ etc.}$$

Since $r^2=(c/2)^2+(r-h)^2$ and $r=(c^2+4h^2)/8h$, we have taking four terms of the series $\dfrac{ch}{2}+\dfrac{c^3h}{6(c^2+4h^2)}+\dfrac{6c^5h^3}{5(c^2+4h^2)^3}+\dfrac{80c^7h^5}{7(c^2+4h^2)^5}=$

$$\frac{140c^{11}h+2912c^9h^3+24576c^7h^5+80192c^5h^7+143360c^3h^9+107520ch^{11}}{210c^{11}+4200c^8h^2+33600c^6h^4+134400c^4h^6+268800c^2h^8+215040h^{10}}=$$

$$\frac{2ch}{3}+\cfrac{1}{\cfrac{15c}{8h^3}+\cfrac{1}{\cfrac{14ch}{15}+\text{ etc.}}}$$

Second convergent $=\dfrac{2ch}{3}+\dfrac{8h^3}{15c}$ and third convergent $=\dfrac{2ch}{3}+\dfrac{56ch^3}{105c^2+60h^2}$.

The value of the fraction lies between these two convergents, which differ only in the term $60h^2$. Now h varies between the limits $c/2$ and c/∞ and as h approaches zero, the two convergents approach each other in value, for each approaches zero. But when $h = c/2$, the difference between the two convergents is largest and by substituting $h = c/2$ in $60h^2$ the third convergent becomes $2ch/3+7h^3/15c$. Taking the arithmetical mean $(8h^3/15c+7h^3/15c)/2 = h^3/2c$ we have for the required approximation $2ch/3+h^3/2c$.

222. A sequel to No. 214. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

The alternate sides of an inscribed quadrilateral intersect in P and Q. Let M be mid-point of PQ and N foot of perpendicular from center of circle upon PQ. Show (1) that MQ = tangent MT, and (2) that PN·NQ = square of tangent NT$'$.

Solution by I. L. Winckler, Cleveland, O.

Let O be the center of the circle and R the radius

Then with the notation of Problem 214, $4\overline{\text{OM}}^2 = 2\overline{\text{OQ}}^2+2\overline{\text{OP}}^2-\overline{\text{PQ}}^2$, since OM is a median of the triangle OPQ.

But $\overline{\text{OM}}^2 = \text{R}^2 + \overline{\text{MT}}^2$; $\overline{\text{OQ}}^2 = \text{R}^2+\overline{\text{QS}}^2$; and $\overline{\text{OP}}^2 = \text{R}^2+\overline{\text{PR}}^2$.

$\therefore$ $4(\text{R}^2+\overline{\text{MT}}^2) = 2(\text{R}^2+\overline{\text{QS}}^2)+2(\text{R}^2+\overline{\text{PR}}^2)-\overline{\text{PQ}}^2$ or reducing, and noticing that $\overline{\text{PQ}}^2 = \overline{\text{PR}}^2+\overline{\text{QS}}^2$ by Problem 214.

$$4\overline{\text{MT}}^2 = \overline{\text{PQ}}^2. \quad \therefore \text{MT} = \tfrac{1}{2}\text{PQ} = \text{MQ}.$$

Again, $\overline{\text{PQ}}^2 = \overline{\text{PR}}^2+\overline{\text{QS}}^2$.

or $(\text{PN} + \text{NQ})^2 = \overline{\text{PR}}^2+\overline{\text{QS}}^2$.

$$\therefore \overline{PN}^2+\overline{NQ}^2+2PN\cdot NQ = \overline{PR}^2+\overline{QS}^2$$

$$\text{or } \overline{OP}^2-\overline{ON}^2+\overline{OQ}^2+\overline{ON}^2+2PN\cdot NQ.$$

$$= \overline{OP}^2-R^2+\overline{QO}^2-R^2.$$

$$\therefore 2PN.NQ = 2\overline{ON}^2-2R^2.$$

$$= 2\overline{NT'}^2+2R^2-2R^2.$$

$$= 2\overline{NT'}^2$$

$$\therefore PN\cdot NQ = \overline{NT'}^2$$

Trigonometry.

223. *Proposed by LeForest McCrosky, Lebam, Wash.*

ABCD represents a field. The length of AB is 295.36 feet; of BC is 379 feet; of CD is 195 feet; of DA is 312 feet. The angle DAB is a right angle. F is some point in DA and E some point in BC. F and E are to be taken at such points that FE will be parallel to AB and that the quadrilateral ABEF will contain one acre. To find the lengths of BE, EF and FA.

I. Solution by W. B. Borgers, Grand Rapids, Mich., and F. A. F. Williams, Aberdeen, Wash.

Draw FE parallel to AB, the diagonal BD, BH parallel to AD, and prolong FE to meet BH at G.

$BD^2 = AB^2 + AD^2$. $\therefore BD = 429.63$.

$\text{Tan } ABD = \dfrac{AD}{AB}$ $\therefore \angle ABD=46^\circ\ 34'\ 9''$.

Having the three sides of the triangle BCD, we find $\angle DBC = 26^\circ\ 59'\text{-}14''$.

$\therefore \angle EBG = 16^\circ\ 26'\ 37''$. Let x = AF or BG.

Then $EG = x \tan EBG = 0.29515x$.

Area $ABEF = \frac{1}{2}(AB + FE)\, x = 43560$ sq. ft.

$\therefore \frac{1}{2}(295.36 + 295.36 - 0.29515x)\, x = 43560$.

$\therefore 0.147575x^2 - 295.36x = -43560$.

$\therefore x = AF = 160.3239$ ft. $EG = 0.29515x = 47.32$.

$BE^2 = BG^2 + GE^2$. $\therefore BE = 167.16$. $FE = AB - EG = 248.04$.

II. Solution by H. E. Trefethen, Kent's Hill, Me.

I. In right triangle DAB find $ABD = 46^\circ\ 34'\ 9.7''$ and then $BD = 429.6295$. In triangle BCD, with three sides known, find $DBC = 26^\circ\ 59'\ 13.8''$. Then $ABC = ABD + DBC = 73^\circ\ 33'\ 23.5''$. Let AD and BC produced meet at V. $AV = 1000.743$, area $ABV = 14779.8$, area $FEV = 147789.8 - 43560 = 104229.8$.

Area ABV : area FEV $= AV^2 : FV^2 = AB^2 : FE^2$. Hence $FV = 840.42$, $AF = AV - FV = 160.323$, $FE = 248.0421$. ($BE = 167.16$) Check $AF\,(AB + FE)\,12 = 43560$.

Credit for Solutions Received.

Algebra 212. E. T. Cushman, A. W. Rich. (2)
Algebra 213. A. W. Rich. (1)
Geometry 214. J. M. Townsend. (1)
Geometry 215. A. W. Rich. (1)
Algebra 217. Orville Price, H. E. Trefethen. (2)
Algebra 218. Orville Price, H. E. Trefethen. (2)
Geometry 219. T. M. Blakslee (2 solutions), R. E. Krug, L. E. A. Ling, L. B. Mullen, Orville Price, A. W. Rich, H. E. Trefethen. (8)

Geometry 220. H. E. Trefethen. (1)
Alegebra 221. C. I. Alexander, T. M. Blakslee, Wm. B. Borgers, P. F. G. Boston, E. T. Cushman, G. E. Congdon (2 solutions), N. F. Davis, F. B. Lyons, A. L. McCarty, Richard Morris, Effie Morse, A. W. Rich, Carrie Sneed, C. C. Spooner, Hilda R. Stice, T. A. F. Williams, I. L. Winckler. (18)
Geometry 222. I. L. Winckler, H. E. Trefethen (2)
Trigonometry 223. T. M. Blakslee (2 solutions), W. B. Borgers, H. E. Trefethen (3 solutions), T. A. F. Williams, I. L. Winckler. (8)
Total number of solutions, 46.

PROBLEMS FOR SOLUTION.

Algebra.

229. *Proposed by E. B. Escott, Ann Arbor, Mich.*

Solve: $x^2-yz=a$ (1),
$y^2-zx=b$ (2),
$z^2-xy=c$ (3).

230. *Proposed by Hilda R. Stice, Petersburg, Ill.*

Solve: $x^2+y=7$ (1),
$y^2+x=11$ (2).

x = 11-y²

231. *Proposed by Richard Morris, New Brunswick, N. J.*

Three men and a boy agree to gather the apples in an orchard for $50. The boy can shake the apples in the same time that the men can pick them, but any one of the men can shake them 25 per cent faster than the other two men and boy can pick them. Find the amount due each.

Geometry.

232. *Proposed by H. E. Trefethen, Kent's Hill, Me.*

On one side of an equilateral triangle describe outwardly a semicircle. Trisect the arc and join the points of division with the vertex of the triangle. Find the ratio of the segments of the diameter.

233. *Selected.*

If AD, BE, CF are the altitudes of the triangle ABC and H their point of intersection, prove

(1) the triangles AFE, CED, and BDF are similar,
(2) $BD\cdot DC = DF\cdot DE = DH\cdot DA$,
(3) circum-circles of triangles AHB, AHC, BHC are equal,
(4) the radius of the circle DEF is one half that of the radius of the circle ABC.

REAL APPLIED PROBLEMS IN ALGEBRA AND GEOMETRY.

At the meeting of the Central Association of Science and Mathematics Teachers in Cleveland in November the committee of the mathematics section on real applied problems in algebra and geometry was asked to continue its work for another year. It was requested especially that the investigation be directed to finding problems of interest to girls. The committee desires the assistance of all interested teachers in this work. All real applied problems of algebra or geometry sent to the chairman, James F. Millis, 330 Webster Ave., Chicago, will be printed in these columns and thus made available for general school use.

ARTICLES IN CURRENT MAGAZINES.

American Forestry for November: "Fundamentals of the Fire Problem," Henry S. Graves; "How the Fires Were Fought," F. A. Silcox; "What Protective Coöperation Did," E. T. Allen; "Forest Fires in Washington and Oregon," C. S. Chapman; "How Telephones Saved Lives," C. J. Buck; "Forest Fires in Washington," Joel Shomaker; "Two Million Dollars' Worth Burned in One Day," General C. C. Andrews; "Fires on the Flathead Forest in Montana," H. H. Chapman; "The Protection of Forests from Fire" (continued); Henry S. Graves.

Popular Science Monthly for December: "The Ilongot or Ibilao of Luzon," David P. Barrows; "Kant and Evolution," Arthur O. Lovejoy; "Some European Conditions Affecting Emigration," Arthur Clinton Boggess; "Genius and Stature," Charles Kassel; "Certain Characteristics of the South Americans of To-day," Hiram Bingham; "When does a Food become a Luxury?" E. H. S. Bailey; "The Paleontologic Record: The Birthplace of Man," S. W. Williston; "The Relation of Paleontology to the History of Man," John C. Merriam; "Two Active Volcanoes of the South Seas," Henry E. Crampton.

Popular Astronomy for December: "On the Limits of Oblateness of a Rotating Planet," Percival Lowell; "An Amateur's Observatory," David E. Hadden; "Some Interesting Spectroscopic Binaries," J. Miller Barr; "Brooks's Periodic Comet," Ralph E. Wilson; "The New Sun Dial or Helio-Chronometer," W. E. Cooke; "The Radial and Tangential Action of Solar Disturbance on Terrestrial Magnetism," T. S. H. Shearmen; "The Determination of Standard Time," C. H. Gingrich.

Photo-Era for November. Among the many splendid illustrations are the following interesting and helpful articles: "The Cadbys: An Appreciation," A. H. Blake; "Hunting and Picture-Making Reminiscences," Charles G. Willoughby; "On the Gullibilty of the Amateur Photographer," Eleanor W. Willard; "Gray Days: Pictorial Suggestions," William Findlay; "A Developer for Black and Brown Tones on Gaslight Papers," D. R. Battles.

Nature Study Review for November: "Uses of the School Garden Harvest," Laura E. Woodward; "Potatoes and Oats as Nature-Study Topics," Alice J. Patterson; "N. E. A. School of Garden Luncheon," Ellen Eddy Shaw; "Weeds," F. L. Holtz.

Sibley Journal of Engineering for November: "Electrification of the New York Central Railroad," Edwin B. Katte; "Some Practical Observations on Ilumination—I," A. G. Rakestraw; "The Mazda Lamp with a Few Common Types of Reflectors," R. Horatio Wright; "Density and Strength of Concrete Blocks Cast in Sand Molds," R. C. Carpenter.

MODEL RURAL SCHOOL ON THE CAMPUS OF A NORMAL SCHOOL.

On the campus of the Kirksville (Mo.) State Normal School a model rural school has been erected for study and observation by students preparing to teach in the country. Pupils are transported to this school from the surrounding farms in covered wagons. The course of study is in line with the most progressive ideas regarding rural schools. The building is modified so as to make large use of the basement and attic.

A BIBLIOGRAPHY OF NORTH AMERICAN GEOLOGY FOR 1909.

The United States Geological Survey has just published as Bulletin 444 a bibliography of North American geology for the year 1909, by J. M. Nickles. This volume covers all publications on the geology of North America that were printed anywhere in the world in 1909, showing the authors, titles, and, briefly, the scope or contents of more than thirteen hundred books and papers. The bulletin includes a voluminous and comprehensive subject index and will be of great value to all geologists and students of geology.

THE IOWA ASSOCIATION OF MATHEMATICS TEACHERS.

The Iowa Association of Mathematics Teachers held its annual meeting in connection with the meeting of the State Teachers' Association at Des Moines Thursday, November 3, at 2:00 p. m.

Miss Laura S. Seals, of the State Teachers' College, read a paper on "A Proper Place for the Graph." Miss Seals illustrated her paper with well prepared charts and showed in a very practical and interesting manner how the graph properly used adds greatly to the effectiveness of teaching and also serves to correlate the mathematics of the secondary course.

Miss Edith Long, of the Lincoln (Neb.) High School, was the next speaker. Her subject was the "Correlation of Mathematics." A complete outline of the course in high school mathematics which has been worked out by Miss Long during the past ten years, was presented and explained in a manner which held the closest attention of the large audience present. There is no question but that Miss Long has accomplished a remarkable piece of work at Lincoln. She has made a plucky fight in the face of opposition sufficient to discourage a less courageous teacher, and is now beginning to reap the fruits of her labors. The course consists of a first year's work in elementary science designed to furnish material for the later study, then three years, six semesters, of splendidly correlated mathematics. Students turned out from this course are more than justifying it in their college work and in technical schools. It is well worth the time of any teacher to visit Lincoln and study the plan in actual operation.

The officers of the association for the coming year are: President, Ira S. Condit, Iowa State Teachers' College; Vice-president, I. E. Neff, Drake University; Secretary-Treasurer, Miss Harriett Solomon, Keokuk High School.

The executive committee was authorized to appoint a committee to prepare a report for next year's meeting on "Ways of Abridging and Enriching the Iowa Public School Course in Mathematics."

ASSOCIATION OF TEACHERS OF MATHEMATICS IN THE MIDDLE STATES AND MARYLAND.

The fifteenth meeting of the Association of Teachers of Mathematics in the Middle States and Maryland was held at the University of Pennsylvania, Philadelphia, on November 26, 1910. The program for the morning session was:

(1) Address of welcome. Edgar F. Smith, Vice-Provost, University of Pennsylvania.

(2) Miscellaneous Business.

(3) Is the Average Secondary School Pupil Able to Acquire a Thorough Knowledge of all the Mathematics Ordinarily Given in These Schools? Isaac J. Schwatt, University of Pennsylvania.

Discussion led by Rev. James J. Dean, Villanova College; Edward D. Fitch, The DeLancey School, Philadelphia; E. B. Ziegler, Conshohocken, Pa.

(4) Election of Officers

Following the morning session the association was entertained at luncheon by the university

The program for the afternoon session was:

(1) Training for Efficiency in Elementary Mathematics. Ernest H. Kock, Jr., Pratt Institute, Brooklyn.

(2) Report of the Committee on Mathematics in Continuation Schools. William E. Breckenridge, chairman, Stuyvesant High School, New York City.

(3) Report of the Committee on Algebra Syllabus. Gustave Le Gras, chairman, College of the City of New York.

The officers for 1910-1911 are: President, William Henry Metzler, Syracuse University, Syracuse, N. Y.; Vice-President, Philip R. Dean, Curtis High School, Staten Island, N. Y.; Secretary, Howard F. Hart, Montclair High School, Montclair, N. J.; Treasurer, Mrs. Clara H. Morris, High School for Girls, Philadelphia, Pa.

Council Members: Paul N. Peck, George Washington University, Washington, D. C.; Susan C. Lodge, Philadelphia Collegiate Institute, Philadelphia, Pa.; Eugene Randolph Smith, Polytechnic Preparatory School, Brooklyn, N. Y.; Isaac J. Schwatt, University of Pennsylvania, Philadelphia, Pa.; Clifford B. Upton, Teachers' College, New York City; Fletcher Durrell, Lawrenceville High School, Lawrenceville, N. J.

HOWARD F. HART, Secretary.

CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The annual meeting of the Central Association of Science and Mathematics Teachers was held at the Case School of Applied Science and the Technical High School, Cleveland, November 25 and 26.

At the general sessions addresses were given by Dr. Dayton C. Miller, Case School of Applied Science, on "Sound Waves—Their Meaning, Registration, and Analysis. With Demonstrations;" and by Dr. Harvey W. Wiley, chief chemist, U. S. Department of Agriculture, Washington, D. C., on "Food Facts Which Every Citizen Should Know." Also valuable reports were presented by the committee on "Fundamentals Common to the Various Sciences and Mathematics," the committee on "Coöperative Experiments in Teaching Science," and the committee on "The Relation of Elementary School Nature Study to Secondary School Science." Important addresses by such men as Dr. David Eugene Smith, Teacher's College, Columbia University, New York, and reports of committees were given before the various sections of the association.

Some of these reports, especially the one on "Fundamentals Common to the Various Sciences and Mathematics," are valuable contributions to education, and it is believed will prove epoch-making in their influence on the teaching of secondary school science and mathematics. They will be found printed in the volume of proceedings, which will be issued in a few weeks.

Among the resolutions adopted by the association were the following:

"That this association should encourage the carrying out of experiments relative to the matter and method of instruction, and that the sections should collect and make available to their members approved methods of testing results, to the end that the resulting conclusions may be definite and reliable.

"That we should strive not only for the 'problem solving' attitude on the part of the pupil, but further we should seek to stimulate him into the 'problem raising' attitude, in order the better to gain the full enlistment of his powers.

"That we believe in the recognition and inclusion within our courses of the practical and applied aspects of the sciences, to insure the proper motivation of the work of the pupil, and to 'bring him quickly and surely to the point where he will respond soundly to really significant stimuli.'"

The next annual meeting will be held in Chicago.

REPORT OF THE MEETING OF THE BIOLOGY SECTION OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

Biology Section—Friday.

The meeting was called to order by the chairman, Mr. Eikenberry, at 1:30. In the absence of the secretary, Miss McAuley was appointed secretary pro tem.

The nominating committee was next appointed—Dr. Coulter, chairman; Prof. Galloway, and Miss McAuley—and instructed to meet at 5 P. M.

Motion was then made and carried that the biology section meet with the earth science section at the close of the program for the paper, "Physical Geography vs. Biology for First Year in High School."

The first paper of the afternoon, "Results of an Experiment with Classes in Zoölogy and Botany," was read by Prof. J. P. Gilbert. Discussion was opened by Dr. Coulter, who suggested that a clearer definition of terms is necessary, the distinction between pure and applied science not being clear. Prof. Galloway urged and approved work in experimentation of the type presented by Mr. Gilbert, and suggested that motivation on the part of the teacher will probably influence such results.

Mr. Clute next presented his paper on "High School Agronomy." A discussion followed in which a number took part, among others, Miss Seaton, Miss Detmers, Mr. McKean, and Miss Beach. The discussion expressed the following views:

1. In the absence of a text, the content of agronomy must be largely made up of experiment and lecture.
2. Time spent on an herbarium, *per se*, is time wasted, but classification acquainting the student with a large number of families and species was urged.
3. The substitution of agronomy for the "spore plants" in a one year course was urged.
4. If only one half year can be devoted to botany, a course based chiefly on structure and function was advocated.

The last thing of the afternoon was the report by Prof. Eikenberry, of one of his experiments with first year students. His report showed the effectiveness of the work given in training the student to draw conclusions from observed facts. The report was graphical. He urged the value of keeping both descriptive records and graphic records of the daily work of the student.

Section adjourned at 4:30.

FAITH MCAULEY,
Secretary pro tem.

Saturday Morning Session.

The session was called to order by the chair at 10 A. M. The report of the committee on the experimental investigation of the teaching of biology was read by the chairman of the committee, Mr. Eikenberry. The report was accepted and motion made and carried, that "the committee be continued and asked to submit a bibliography of articles relating to the experimental field."

The report of the nominating committee was next presented, the nominees being Miss Rosseau McClellan, chairman; Mr. Lucas, vice-chairman; Miss McAuley, secretary. The nominees were elected.

The remaining time was devoted to the presentation of experimental reports. First of these was presented by Miss McAuley and dealt with the

content and appeal of the first year science course. Some discussion followed, introduced by Miss Dawson, who indicated briefly the nature of the biology work in the Cleveland schools. Identification of trees, forestry problems, insect types and problems, bacteriology, and elementary problems in agriculture were indicated as topics handled. Throughout the work the viewpoint is distinctly the economic.

In the absence of the authors, Mr. Eikenberry next presented reports by Mr. Lucas and Mr. Finley. Mr. Lucas's report dealt with the results of segregating the classes in zoölogy. Mr. Finley's report presented results of experiments having as their problem the type of interest paramount in the successive years in the grades.

The session closed with these reports and adjourned at 11:30.

FAITH MCAULEY,
Secretary pro tem.

REPORT OF THE MEETING OF THE MATHEMATICS SECTION OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS.

The section met at the Technical High School, Cleveland, Ohio, Friday and Saturday, November 25 and 26. The following business was transacted:

The question of the continuation of the Committee on Fundamentals was left to the chairman and vice-chairman for next year.

The report of the Committee on Uniform Notation in Mathematics and the Sciences, was received and the committee continued.

The report of the Committee on Results was received and the committee continued.

The Committee on Real Applied Problems was continued with a view to the continuation of the applied problem department in SCHOOL SCIENCE AND MATHEMATICS.

The chairman was authorized to appoint someone to confer with the editors of SCHOOL SCIENCE AND MATHEMATICS concerning reprints of the applied problems that have appeared in the magazine during the current year.

The appointment of a Publicity Committee was ordered.

It was decided that all reports of committees appointed a year in advance, must be printed and distributed before the meeting at which these reports come up for adoption and that they must be discussed at that meeting.

Mr. J. F. Millis presented a bill of $8.00 for reprints of the Report of the Committee on Real Applied Problems. The bill was allowed and ordered paid from the money collected at the 1909 meeting.

The following officers were elected for the coming year: Chairman, Mr. R. L. Short, Technical High School, Cleveland, Ohio; Vice-Chairman, Mr. I. S. Condit, Iowa Teachers' College, Cedar Falls, Iowa; Secretary, Miss Marie Gugle, Central High School, Toledo, Ohio.

The papers presented are reported below:

The Fundamental Reasons for Teaching High School Mathematics. Professor David Eugene Smith, New York City. Professor Smith showed that the teaching of secondary mathematics could not be justified on any ground of immediate utility. He considered the distinctive features of mathematical thought and of mathematics teaching during certain epochs of the world's progress and discussed the prospects for the future showing the need at the present time for the kind of training that mathematics gives. He said that from the standpoint of the pupil the immediate aim in the study of mathe-

matics is pleasure, but that for the race the ultimate aim far transcends this. In particular, profit is to be derived from the study of geometry because (1) it is an exercise in logic, (2) it gives exercise in accurate and precise thought and expression, (3) it leads to an appreciation of the dependence of one magnitude upon another, (4) it cultivates space intention and an appreciation of and control over forms existing in the material world and (5) because of its application to mensuration. While warning against the danger of false applications of high school mathematics, Professor Smith stated his appreciation of the value of all genuine applications particularly of a local nature.

Report of the Committee on Fundamentals. I. S. Condit, Cedar Falls, Iowa, chairman. In the absence of Mr. Condit, the report was read by Mr. R. L. Short of Cleveland. The report reviewed the geometry report of 1906, the algebra report of 1907, the unification report of 1908, and the preliminary report of the Committee on Real Applied Problems, presented in 1909. The report stated that the fundamentals in algebra are (1) training in generalization and (2) formulation of knowledge for ready use in later work, that the aim in the study of geometry is training in logical reasoning, and that the fundamentals in geometry are the body of definitions and assumptions which are determined by the necessities encountered in making geometry a suitable pedagogical instrument rather than in building up a pure science. The report closed with a statement of the pedagogical principles that must underly the teaching of secondary mathematics.

Report of the Committee on a Uniform System of Notation in Mathematics and the Sciences. Mr. L. P. Jocelyn of Ann Arbor, Mich., chairman. Among the recommendations made by the committee are the following: That 0 be read *zero;* that decimal fractions be called decimal fractions rather than decimals and that in arithmetic, at least, they be read as fractions; that $\frac{a}{b}$ be read *a divided by b*, especially in primary and secondary schools; that the fundamental operations be performed with abstract numbers only; that $a^{\frac{1}{4}}$ be read *a exponent* $\frac{1}{4}$; that ratios be written in the fractional form; that "*destroy each other*" be used for "*cancel*" in operations involving addition and subtraction; that *f* be used instead of *a* for the first term of progressions. The report discussed the definitions of positive and negative numbers, the use of the various symbols of operation, the various definitions of division, the possibility of the four cases, \$6÷\$2, \$6÷2, 6÷\$2, and 6÷2, the interpretation of exponents in the fractional form, the use and the reading for the various signs of relation, aggregation, continuation, and deduction, the treatment of proportion and the names for various operations involving proportion.

The *discussion* that followed called attention to certain important omissions, as for example, $x=ay$ instead of x varies as or is proportional to y.

Report of the Committee on Results. Professor C. E. Comstock, Peoria, Ill., chairman. In the absence of Mr. Comstock the report was read by Mr. H. T. McMyler of Cleveland, a member of the committee. This committee is a subcommittee of one appointed a year ago by the association to consider the scientific study of teaching problems and especially impersonal tests of methods and results. As the general committee has but recently promulgated its plans, the subcommittee has not had sufficient time to prepare anything beyond a preliminary report of progress. The report referred to the questionaire recently sent to members of this section and contained a brief summary of replies. These replies seemed to indicate that teachers test the efficiency of methods by testing pupils, and that there is little conscious effort to judge of the efficiency of one method by a comparison of the results

of different methods. The report then discussed the difference between testing pupils and testing methods, and general aims and methods in mathematics teaching.

Aims and Tests in Algebra, a Discussion of the Report of the Committee on Results. Mr. H. L. Terry, Madison, Wis. Mr. Terry discussed three great values in algebra as an instrument in mathematical operations. These are: first, its value as an instrument in economizing thought by giving power to think in symbols rather than in complicated English expressions; second, its value as an instrument of generalization; third, its value as an instrument for simplifying calculations. Mr. Terry stated that a thorough mastery of the algebra language is essential to any real appreciation of these values and that this mastery is to be attained only by repeated exercises in translation, by frequent use of problems and by the elimination of much of the difficult and complicated work in texts.

Discussion of the Report of the Committee on Results. Mr. C. C. Carlton, Superintendent of Schools, Medina, Ohio. Mr. Carlton was very emphatic in declaring that the good work done by the association is far too limited in extent. Every science teacher and every mathematics teacher should secure the annual proceedings and profit by the abundance of material offered in the association's official organ. The difficulty with teachers is not that they are confused by the multitude of suggestions as the report of the committee states, but that they are entirely ignorant of these suggestions. Mr. Carlton discussed means of testing the efficiency of aims and methods and, while commending the suggestions of the committee, gave as the final basis for judgment, the response of the pupil—not in his ability to memorize—but in his thorough mastery that is in his vital interest.

Second Report of the Committee on Real Applied Problems. Mr. J. F. Millis, Chicago, Ill., chairman. The committee had been continued a second year to give an opportunity for testing the adaptability of the problems that have appeared for the last two years in SCHOOL SCIENCE AND MATHEMATICS, to secondary school uses. Mr. Millis called attention to the main features of the report which was published in the November number of SCHOOL SCIENCE AND MATHEMATICS.

Discussion of the Report of the Committee on Real Applied Problems. Miss Marie Gugle, Toledo, Ohio. Miss Gugle discussed the meaning of the words *real* and *practical* as applied to problems and showed that practical problems are not always real to the student because (1) they may require a technical knowledge that he does not possess and (2) they may deal with conditions of which he knows nothing. Fundamental principles in mathematics must be known before they can be applied to any kind of problem, for that problem is too difficult whose conditions the pupil cannot translate into general conditions. Caution is necessary therefore that the particular problem do not lead too far from the general. In the second place many of the problems that have appeared are too technical for general use in the average high school class of mixed pupils. The real problem committee has a more difficult task before it than was first supposed, for it is not sufficient that they gather concrete problems that are real to men in certain occupations; these problems must be real also to boys and girls of high school age. The work of the committee is to be highly commended, but a still more thorough and extended testing of the adaptability of individual problems for use in the mixed classes of non-technical high schools, is needed.

Discussion of the Report of the Committee on Real Applied Problems. Mr. W. E. Stark, New York City. Mr. Stark was very enthusiastic in his congratulations to the section upon the report of the committee and the general success of the undertaking, the most valuable part of which he believes to be the collection of problems itself and the demonstration of the possibil-

ity of improving the collection through coöperative effort. He discussed three considerations bearing upon the value of real problems. First, no one set of problems has superior merit under all conditions. Their use requires differentiation. The problems given should be suited to the needs of individual pupils that each may be stimulated to his best effort. Second, the success of the problems will depend in large measure on the teacher's attitude toward them. Illustrations introduced incidentally out of a full experience, are often the most effective. It is to be regretted that most high school teachers have made no study of applied mathematics. Thirdly, a knowledge of the facts and relations of formal mathematics is of little value unless they are so associated in the pupil's mind with other ideas that they will lead to a modification of his thinking. Real problems should be used, therefore, in such a way as to make the pupil's conception of mathematical principles and methods broader in scope, to give him some recognition of the dependence of our present possessions in the way of regularly laid out cities, of buildings, machinery, furniture, and the like, upon mathematics, and to arouse in him some respect and admiration for this wonderful possession of the people of our time. MABEL SYKES, Secretary.

REPORT OF THE PHYSICS SECTION SESSION AT THE MEETING OF THE CENTRAL ASSOCIATION OF SCIENCE AND MATHEMATICS TEACHERS HELD AT CLEVELAND, NOVEMBER 26 AND 27, 1910.

After an inspection of the new Technical High School of Cleveland, the physics teachers went to the third floor where an opportunity was given them to examine the exhibits of physical apparatus made by several manufacturers. At two o'clock the physics section convened in an adjoining room under the chairmanship of Mr. C. E. Spicer of Joliet, Ill.

The chair appointed as a nominating committee Mr. Turton of Chicago; Mr. Macauley, St. Charles, Ill.; and Mr. C. M. Bronson, Toledo, O.

In the absence of Professor Reed the program was opened by Mr. Paul G. W. Keller, principal of the high school, Appleton, Wis., with a paper describing an experiment in that school in which the subject of physics is taught in two short courses instead of one long course. One course consists of a general survey of the subject in its more elementary aspects intended for those who can afford only a portion of a year for physics. The second course is more intensive and extended intended for those having more time for the subject and greater interest in it. The plan has been in operation for too short a time to justify any sweeping conclusions, but it promises great success.

The paper created considerable interest and at its close many questions were asked in regard to the plan. Professor J. O. Reed, of the University of Michigan, was the next speaker. He gave an address replete with interest on the subject, "Teaching of Physics in High School and University." This address to be appreciated needs to be read in full, and hence no analysis of it will be attempted here. At its conclusion the section on motion of Mr. Tower of Chicago gave Dean Reed a unanimous vote of thanks for his inspiring address.

The paper on the *Freeport School and Shop Coöperative Course* was, in the absence of its author, Mr. Fulwider of Freeport, Ill., read by Mr. H. T. McMyler of Cleveland. In the discussion following the paper, Mr. Terry asked if anyone present had had any experience with such a course. Mr.

McMyler knew something of the work at Freeport. He stated that no fee was asked of the pupils at the shops but that a bonus was paid to them. Mr. Terry: "If the factory is unionized, how can the pupil get in?" Mr. McMyler: "There is no opposition at Freeport. It was reported that there was opposition at Fitchburg, Mass. Lewis Institute it was said is trying the plan." Mr. Turton: "In Chicago two years in a vocational school count on apprenticeship." Mr. Spicer: "Labor organizations favor this more." Mr. Terry criticised the academic part of the course given the boys as exceedingly narrow. Mr. McMyler: "It brings into the high school those who would not otherwise be there; even a narrow course is better than none."

Mr. Twiss, Columbus: "These boys are promised transfers from one machine to another; it is said the promise is often not fulfilled by shop foremen."

Professor A. D. Cole, Ohio State University, was the next speaker of the afternoon. His subject was, "The Relation Between Physics and Manual Training." He gave an interesting account of his early experience in teaching in a small college with little apparatus, but possessing some tools. He instituted work on the part of the students in making apparatus which developed into a permanent and very popular course in the college.

Saturday forenoon the physics and chemistry sections held a joint session, presided over by Mr. Spicer and Mr. A. L. Smith, Englewood High School.

The nominating committee of the chemistry section reported the following nominations: Chairman, Frank B. Wade, Shortridge High School, Indianapolis, Ind.; Vice-Chairman, Howard W. Adams, State Normal School, Normal, Ill.; Secretary, Miss Lillian Kurtz, Medill High School, Chicago.

The secretary was directed to cast the ballot of the section for those named and it was so done.

The following officers were elected for the physics section: Chairman, H. L. Terry, State High School Inspector, Madison, Wis.; Vice-Chairman, V. D. Hawkins, Technical High School, Cleveland, O.; Secretary, Willis E. Tower, Chicago.

The first paper of the forenoon by Mr. R. O. Austin, Central High School, Columbus, outlined a first-year course in science. Mr. Austin gave a very complete list of the experiments used in the course, including work in biology, chemistry physics, and domestic science. Many of them were home experiments, some of them lecture-table experiments.

In the discussion which followed the paper, Mr. Morrison outlined the work at Pittsburg, Pa. There all experiments except home and lecture-table experiments had been dropped. Mr. Spicer: "It makes a difference to the physics and chemistry teacher what science has preceded their work. I believe we should have four years of consecutive science."

Physics in Technical Schools was the subject of a paper by Mr. V. D. Hawkins of Cleveland. This paper caused considerable discussion. Mr. Nye: "How is laboratory work begun?" Mr. Hawkins: "By use of micrometer and measuring instruments. At first we attempted to dispense with this, but found that special instruction in the use of these instruments necessary. We give much attention to the study of efficiency of machines."

The last paper of the forenoon was given by Mr. W. E. Tower of Chicago on, "An Experiment: The Teaching of Physics in Segregated Classes."

The discussion of the paper brought out the general opinion that some teachers succeed best with girls, some with boys, and others with mixed classes.

Mr. Tower reported that girls were much more free to ask questions in girls' classes.

Mr. Bronson: "We are trying the experiment in Toledo. I agree with Mr. Tower in his conclusions."

Miss Pancost: "I prefer mixed classes. Girls learn the text better than boys."

On motion of Mr. Keller, Mr. Tower was asked to continue the work and report at the next meeting. An amendment offered by Mr. Adams that Mr. Tower be permitted to add others to the committee who may be trying the same experiment was accepted and the motion carried.

The meeting then adjourned.

C. F. ADAMS, Secretary

LITERARY NOTE.

The "Portuguese Grammar," by Professor John C. Branner of Stanford University, which Messrs. Henry Holt & Co. will issue late in October, appears at possibly the most interesting period in the history of that little country. It is, as far as known, the only grammar of that language in English.

Pupils who used to have to go through the hackneyed list of the French classic drama must envy the present generation who seem to get the newest and best. Coppee's highly picturesque play about a parracide, who was noble in more senses than one, entitled "Pour la Couronne," which has been done here in English by Henry Vroom, and in England by such famed actors as Forbes Robertson, and Mrs. Patrick Campbell, will be issued in the original French text, with English introduction and notes by Dr. R. L. Hawkins of Harvard, by Messrs. Henry Holt & Co., on October 29.

COAL MINING IN ARKANSAS IN 1909.

Conditions Adverse to Profitable Mining.

Coal mining in Arkansas in 1909 showed no marked changes in conditions from those of 1908. In neither year was the business satisfactory to the operators or the miners. Competition with petroleum and natural gas, resulting from the development of the Louisiana and mid-continental fields, has adversely affected the market for Arkansas coal and since 1907 has reduced the price from $1.68 to $1.48 a ton. In addition to this, whatever benefit might have been gained in 1909 by the recovery from the depression of 1908 was largely offset by a drought which lasted from the first of June to the middle of November. This drought not only created a crop shortage in the state which affected the demand for fuel but caused great scarcity of water at the mines and raised the cost of production by increasing the expense of providing water for the boilers.

Chauvenet has attempted to prepare metallic thorium free from oxide by using metallic lithium in place of sodium for reducing thorium chloride. By heating the mixture in an iron boat in a quartz tube from which air was carefully excluded, a product containing 96 to 96.6 per cent of metallic thorium was obtained. This, however, contained 3.2 per cent of thorium oxide which could not be removed. A purer product (96 to 97 per cent of metallic thorium) was prepared by heating thorium hydride (obtained by heating together thorium chloride and lithium hydride to 600 deg. C.) in a vacuum of 10 millimeters. The metallic borium thus obtained is black; it is not oxidized by air or by pure oxygen at atmospheric pressure, but is acted upon by oxygen under pressure. It takes fire with a luminous flame in fused potassium chlorate. It combines with chlorine to form thorium chloride, $ThCl_4$, with the liberation of 339.43 calories.

BOOKS RECEIVED.

Domesticated Animals and Plants, by Eugene Davenport, Dean of the College of Agriculture, Director of the Agricultural Experiment Station, and Professor of Thremmatology in the University of Illinois. 8vo., cloth. 321 pages. Illustrated. List price, $1.25. Ginn & Company, Publishers, Boston, New York, and Chicago.

An Introduction to Zoölogy, by Robert W. Hegner, University of Michigan. xii+350 pages. 15x22 cm. $1.90 net. The Macmillan Company, New York.

American Men of Science, edited by J. McKeen Cottell, editor of Popular Science Monthly. xiii+596 pages. 20x26 cm. $5.00. The Science Press, New York.

The Story of Great Inventions, by Elmer Ellsworth Burns, Instructor in Physics, Medill High School, Chicago. xiii+249 pages. 15x20 cm. Many illustrations. Harper and Brothers, New York.

Physics in Small High School, by Eastern Association of Physics Teachers. 28 pages. Dunbar-Kerr Company, Malden, Mass.

Anharmonic Coördinates, by Lieut.-Colonel Henry W. L. Hime. Pp. xiv+127. 15x22 cm. Longmans, Green & Co., New York.

BOOK REVIEWS.

The Mechanics of Writing, by Edwin C. Woolley, Ph. D., Department of English, University of Wisconsin, Author of Handbook of Composition. Cloth. xxxi+396 pages. D. C. Heath & Co. 1909. Price, $1.00.

This is a companion book to the Handbook of Composition reviewed in the November number. It is a compendium of rules regarding manuscript arrangement, spelling, the compounding of words, abbreviations, representation of numbers, syllabication, the use of capitals, italics, punctuation, and paragraphing. It should be in the hands of every student and teacher of English. It is a most invaluable handbook for stenographers and writers. It puts punctuation on a logical basis. C. M. T.

Plane and Spherical Trigonometry, by D. A. Rothrock, Ph. D., Professor of Mathematics in Indiana University. Pp. xi+256. Price, $1.00. 1910. The Macmillan Company.

The author places emphasis on drill work in the trigonometric identities, on the applications of trigonometry to practical problems, and on approximate calculations by means of natural functions. The use of squared paper to secure approximate results and to give a rough check is to be commended. H. E. C.

Analytic Geometry, by N. C. Riggs, M.S., Assistant Professor of Mathematics, Carnegie Technical Schools, Pittsburg, Pa. Pp. xi+294. Price, $1.60. 1910. The Macmillan Company.

That the author has written this book from a new viewpoint and has made an excellent selection of new material is evident as one looks through the volume. Much less space is given to the straight line and to the conic sections than is given in most text-books; and the chapters on trigonometric and exponential functions, parametric equations, empirical equations, maxima and minima, and graphical solution of equations furnish desirable work for engineering students.

The method and notation of calculus have been used in finding the tangents to curves and in solving maxima and minima problems. Chapter XVI, Empirical Equations, gives a simple exposition of a topic which has not been sufficiently developed in American text-books. Fifty pages are given to analytic geometry of space; and the discussion is ample for the purposes of calculus. This book merits a careful examination and will undoubtedly be adopted in many colleges and engineering schools. H. E. C.

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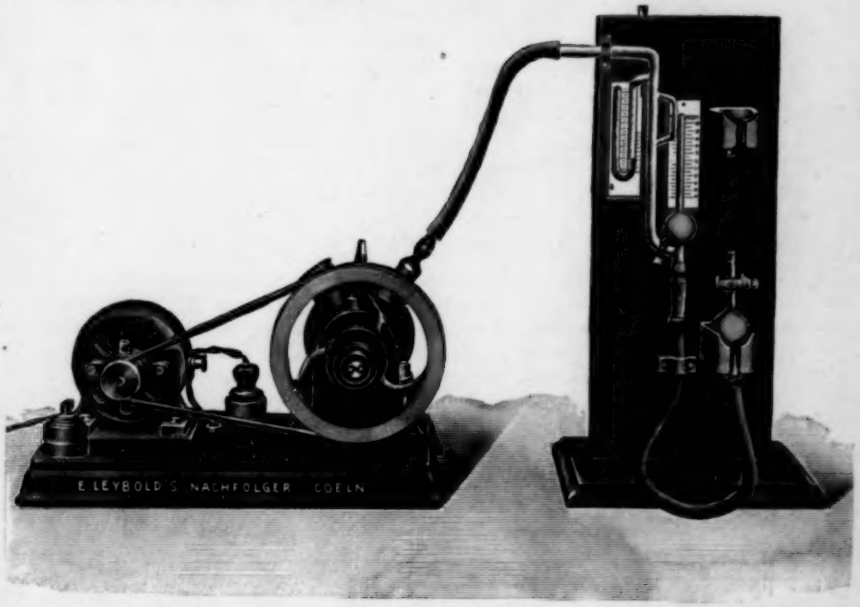

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Elementary Analysis, by Percy F. Smith, Ph.D., and William A. Granville, Ph.D., The Sheffield Scientific School of Yale University. Pp. x+223. 1910. Ginn and Company.

This book aims to give a simple and direct discussion of those parts of analytic geometry and calculus which are of importance to students of natural sciences. The text is in part identical with portions of the authors' text-books already published; this material, however, has been revised or rewritten to meet the needs of students who want to get a working knowledge of analytic geometry and calculus. Chapters V and VI give methods of plotting curves and solving maxima and minima problems by the use of graphs. For a short course this text-book offers some excellent work.

H. E. C.

Bookkeeping and Business Practice, by Wallace H. Wigham and Oliver D. Frederick. D. C. Heath & Co., Boston.

This practical development of the accountant idea, not only embodying the class room experience of two veteran commercial teachers but instinct with their pedagogical aim, exacts genuine attention from secondary school instructors. Explicit directions, exercises and questions, simple emphasis, and the exclusion of extraneous matters combine with modernity and flexibility of arrangement. Business forms are introduced after an undeniably thorough exposition of theory. An advanced volume is in preparation, whose novel feature will be detailed household accounts, and whose varied selection of subject promises to elaborate the definite and methodical foundation of the present book. A. A.

First Year Mathematics for Secondary Schools, by George W. Myers, Professor of the Teaching of Mathematics and Astronomy, College of Education of the University of Chicago, and the Instructors in Mathematics in the University High School. Third Edition. 1909. Pp. xii+365. Price, $1.00 net, $1.13 postpaid. The University of Chicago Press.

This volume aims to present in teachable form a combination of the more concrete and easier portions of the first courses in both algebra and geometry. It covers the work of the first year of secondary schools, and places the chief emphasis on algebra while including many related fundamental notions and principles of geometry.

The processes of addition, multiplication, and so on are touched upon several times and in each case they are connected with real problems (the sides of polygons, areas, angles, leverages, and so on), so that the pupil may from the beginning realize that the literal symbols, the symbols of operations and the equations have to do with real things and with the solution of real problems.

The present volume is the result of four years' experimentation in the class room of the University High School; and it is claimed that it will work easily and effectively in any high school. No doubt there are many schools where it is possible to adopt this book and the *Second-Year Mathematics* at once. Teachers who are interested in the improvement of the teaching of mathematics should examine these books. H. E. C.

Wentworth's Plane Geometry. Revised by George Wentworth and David Eugene Smith. Pp. 287. Ginn & Co., 1910.

A more concrete introduction in the form of a number of practical problems and interesting exercises in drawing, tending to impress the pupil with the usefulness of his new subject of study; an earlier and freer use of easy algebraic applications; the reduction of the number of propositions so as to

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include only the great basic theorems and problems; the placing of some topics commonly regarded as less important, especially symmetry and maxima and minima, in the appendix; and the systematic grouping, usually in full pages, of the greatly increased number of simple exercises, together with the omission of the difficult puzzle found too frequently in the American text-book, appear to be the principal changes wrought in the well-known Wentworth Geometry, in this latest edition. Nearly all of these changes are in accordance with the prevailing ideas regarding the teaching of elementary geometry, and they have resulted in a substantial betterment of what was already one of the best of American mathematical texts.

As stated in the preface, in undertaking this revision, "every effort has been made not only to preserve but to improve upon the simplicity of treatment, the clearness of expression, and the symmetry of page that have characterized the successive editions of the work." Many of the results of this effort may escape the attention of the casual reader; but a more careful perusal will show that little is left to be desired in the way of simplicity, clearness, and directness.

"The proofs have been given substantially in full, to the end that the pupil may always have before him a model for his independent treatment of exercises." This is not fully in accordance with the prevailing fashion, but it is believed that the experience of every observing teacher of large classes of young pupils in this subject confirms the soundness of this view. Perhaps ten per cent of the pupils in the average high school do not need to have the "model always before them;" but the remaining ninety per cent do need it, else they never form the "geometric habit," and acquire little command of the subject. It should be recognized also that "originals" afford ample scope for the exercise of all the ingenuity and native ability of the brightest of our pupils.

A brief history of geometry will engage the attention and interest of the inquiring pupils.

The mechanical execution of the book is most excellent, and by its clear, open, not over-congested page, affords a striking contrast to some of the mathematical texts that have recently come from the press.

DUANE STUDLEY.

Text-Book of Elementary Zoölogy for Secondary Educational Institutions. By Thomas Walton Galloway, Ph.D., Professor of Biology in the James Millikin University, Decatur, Ill. Pp. xx+418. 1910. Price, $1.25 net. P. Blakiston & Co., Philadelphia.

This is a notable book which probably marks the beginning of a new era in the teaching of the biological sciences. The author has recognized the fact that forces are now working which will greatly modify the present methods in zoölogy or do away with the subject altogether. But it is better to let the author speak for himself, as he has done in the preface of the book. He divides teachers into three classes: "(1) those presenting pure morphological and physiological (evolutionary) zoölogy; (2) those who make the scientific or evolutionary view primary and yet supplement this liberally with the economic emphasis; and (3) those who insist that the approach and the emphasis shall be primarily economic and that the scientific viewpoint shall be purely supplementary to this."

The author frankly sympathizes with the latter view and says truly, "Of all sciences, our own—biology—should certainly be the first to insist that its content and method should be so ordered as to enable the youth to secure sound and practical adjustment to actual life relations. * * * We are training men and women, and not zoölogists; we are helping candidates for life, not candidates for college entrance."

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We shall see, now, how he carries out his creed. The first twelve chapters are introductory and specials; then follow fourteen chapters treating of the various phyla of animals in the ordinary evolutionary way. Finally, the book concludes with four chapters on man, the distribution of animals, the evolution of animals, and the economic importance of animals. Some of the most noteworthy features may be mentioned. There is a preliminary field exercise, making this an introduction to the work in zoölogy, followed by a chapter on classification and a general survey of the animal kingdom, with a key to the phyla. This is to enable the student to place the animals he collects. The idea of introducing the year's work with field exercises is a good one. The next step is to learn what life is by studying various forms of matter and contrasting them, passing by easy stages to the forms of life, and finally to a consideration of plants and animals and man's dependence on them. Four chapters are given to this work, intended to occupy three to five weeks' time. In the opinion of the reviewer, too much time is given to this phase of the work—more than the importance of the topic warrants, even though the author makes use of the study to teach methods of laboratory study and reasoning from data.

The pupil is now considered ready to study a representative animal—locust or crayfish. But his training is not done, for four chapters on functions, development, and ecology follow before the evolutionary series is taken up. As a means for relating the study to human interest, numerous topics for library work are given. These are all very suggestive and good. There is also a set of keys to orders and classes and to common species. This is the first time such keys have been given in a text-book in a form simple enough for a secondary school pupil to use. It is one of the most valuable features of the book. Another interesting innovation is the introduction of questions on the illustrations. This would seem sensible and will no doubt be helpful. Throughout the book there is much skill and sound pedagogy in arrangement and presentation of topics.

As to whether the book will be successful in the schoolroom in actual use, the writer is not convinced. There are faults that cannot be overlooked. The book combines text and laboratory instruction; in fact, it comes nearer being a laboratory manual than a text-book. I think this a serious fault. The use of a text and its illustrations ought to be under the control of the teacher, but with the combined book this is impossible. The text-book ought to be a reference book. A more serious fault is the prolixity of questions and a style too advanced for second year high school pupils. The author is a college man, and though very many college professors are authors of texts for high schools, the number that have written really successful books is very small. After years spent in teaching college students these men cannot realize the fact that the second year high school has not learned to reason or follow the close reasoning of another. Neither have they acquired a vocabulary sufficient to cope with the flow of most of these writers. That the author of this book has not gotten away from the style he uses for freshmen in college is not to be wondered at. He will need to try two or three times as others recently have done before he can hope to succeed.

A very curious fault is to be noted in some places, where the author takes the didactic attitude not of lecturer to pupils in high school but of lecturer to fellow teachers. This comes, of course, out of the author's well-known activity in presenting papers on secondary education before the Central Science and Mathematics Association. There is no doubt, however, of the value of the contribution to science the author has made with this book, and it will rank with Ganong's "Teaching Botanist" as a book of methods. It surely should be in the hands of every zoölogy teacher. W. W.